# Renormalization of the quantum chromodynamics with massive gluons

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In our previously published papers, it was proved that the chromodynamics with massive gluons can well be set up on the gauge-invariance principle. The quantization of the chromodynamics was perfectly performed in the both of Hamiltonian and Lagrangian path-integral formalisms by using the Lagrangian undetermined multiplier method. In this paper, It is shown that the quantum theory is invariant with respect to a kind of BRST-transformations. From the BRST-invariance of the theory , the Ward-Takahashi identities satisfied by the generating functionals of full Green functions, connected Green functions and proper vertex functions are successively derived. As an application of the above Ward-Takahashi identities, the Ward-Takahashi identities obeyed by the massive gluon and ghost particle propagators and various proper vertices are derived and based on these identities, the propagators and vertices are perfectly renormalized. Especially, as a result of the renormalization, the Slavnov-Taylor identity satisfied by renormalization constants is natually deduced. To demonstrate the renormalizability of the theory, the one-loop renormalization of the theory is carried out by means of the mass-dependent momentum space subtraction scheme and the renormalization group approach, giving an exact one-loop effective coupling constant and one-loop effective gluon and quark masses which show the asymptotically free behaviors as the same as those given in the quantum chromodynamics with massless gluons.

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## I. INTRODUCTION

According to the conventional concept of quantum chromodynamics (QCD), in order to keep the Lagrangian to be gauge-invariant, the gluons must be massless. On the contrary, in many previous investigations of glueballs, an effective gluon mass was phenomenologically introduced so as to get reasonable theoretical results [1-4]. The gluon mass was supposed to be generated dynamically from the interaction with the physical vacuum of the Yang-Mills theory [3] or through strong gluon-binding force [5]. Apparently, these arguments would not be considered to be stringent and logically consistent with the concept of the ordinary QCD. In our previous papers [6-8], it was argued that the QCD with massive gluons, as a non-Abelian massive gauge field theory in which the masses of all gauge fields are the same, can actually be set up on the principle of gauge-invariance without need of introducing the Higgs mechanism. The essential points to achieve this conclusion are as follows. (a) The gluon fields must be viewed as a constrained system in the whole space of vector potentials and the Lorentz condition, as a necessary constraint, must be introduced from the beginning and imposed on the massive Yang-Mills Lagrangian; (b) The gauge-invariance of a gauge field theory should be generally examined from its action other than from the Lagrangian because the action is of more fundamental dynamical meaning than the Lagrangian. Particularly, for a constrained system such as the gluon field, the gauge-invariance should be seen from its action given in the physical subspace defined by the Lorentz condition because the fields exist and move only in the physical subspace; (c) In the physical subspace, only infinitesimal gauge transformations are possibly allowed and necessary to be considered in the examination of whether the theory is gauge-invariant or not; This fact was clarified originally in Ref. [9]; (d) To construct a correct gauge field theory, the residual gauge degrees of freedom existing in the physical subspace must be eliminated by the constraint condition on the gauge group. This constraint condition may be determined by requiring the action to be gauge-invariant. Based on these points of view, it is easy to prove that the QCD with massive gluons established in our previous papers [6-8] is gauge-invariant.

In Refs. [6-8], the quantization of the QCD with massive gluons (will be called massive QCD later on) was Lorentz-covariantly performed in the both of Hamiltonian and Lagrangian path-integral formalisms by employing the Lagrange undetermined multiplier method. In this paper, it will be shown that the quantum theory has an important property that the effective action appearing in the generating functional of Green functions is invariant with respect to a kind of BRST-transformations [10]. Thus, the theory is set up from beginning to end on the basis of gauge-invariance principle. From the BRST-symmetry of the theory, we will derive various Ward-Takahashi (W-T) identities [11-17] satisfied by the generating functionals of Green functions and proper vertex functions. These W-T identities are of special importance in proofs of unitarity and renormalizability of the theory [18]. Furthermore, from the W-T identities obeyed by the generating functionals, we will derive W-T identities satisfied by the massive gluon propagator, the massive ghost particle propagator, the gluon three-line proper vertex, the gluon four-line proper

vertex and the quark-gluon proper vertex which appear in the perturbative expansion of S-matrix elements. Based on these W-T identities, the aforementioned propagators and vertices will be perfectly renormalized. As a result of the renormalizations, the Slavnov-Taylor (S-T) identity satisfied by the renormalization constants [19, 20] will be derived and shown to be formally the same as that given in the QCD with massless gluons (will be called massless QCD hereafter). This identity is much useful for practical calculations of the renormalization by the approach of renormalization group equation [21-23]. It should be mentioned that in the previous literature [16,24-28], the massive non-Abelian gauge field theory without involving Higgs bosons in it was not considered to be renormalizable and/or unitary. This conclusion was drawn from the theories which were not established correctly because the unphysical degrees of freedom involved in the theories are not eliminated at all by introducing appropriate constraint conditions. In our theory, the unphysical degrees of freedom appearing in the theory, i.e., the unphysical longitudinal components of vector potentials for the gluon fields and the residual gauge degrees of freedom existing in the subspace defined by Lorentz condition are respectively eliminated by the introduced Lorentz condition and the ghost equation which acts as the constraint condition on the gauge group. This guarantees that the massive QCD established in our previous papers is not only unitary, but also renormalizable. To demonstrate further the renormalizability of the theory, in this paper, the one loop renormalization will specifically be carried out by means of the mass-dependent momentum space subtraction scheme and the renormalization group equation (RGE), giving an exact one-loop effective coupling constant and one-loop effective quark and gluon masses without any ambiguity.

The arrangement of this paper is as follows. In section 2, we will derive the BRST-transformations under which the effective action of the massive QCD is invariant. In section 3, we will derive the W-T identities satisfied by various generating functionals. In section 4, to illustrate applications of the above W-T identity, the W-T identities obeyed by the massive gluon propagator and ghost propagator will be derived and the renormalization of these propagators will be discussed. In sections 5-7, the W-T identities obeyed by the gluon three-line vertex, the gluon four-line vertex and the quark-gluon vertex will be derived and the renormalizations of the vertices will be discussed, respectively. Section 8 serves to derive the one-loop effective coupling constant and the effective gluon mass. Section 9 is used to derive the one-loop effective quark masses. In the last section, some conclusions and discussions are made. In Appendix, the W-T identities will be given by an alternative derivation.

## II. BRST- TRANSFORMATION

In Ref. [7], the QCD with massive gluons is set up by starting from the Lagrangian

$$\mathcal{L}=\bar{\psi}\{i\gamma^{\mu}(\partial_{\mu}-igT^{a}A_{\mu}^{a})-m\}\psi-\frac{1}{4}F^{a\mu\nu}F_{\mu\nu}^{a}+\frac{1}{2}M^{2}A^{a\mu}A_{\mu}^{a} \tag{2.1}$$

where $\psi(x)$ denotes the quark field function , $\bar{\psi}(x)$ is its Dirac-conjugate, $T^a=\lambda^a/2$ are the color matrices (the generators of gauge group $SU(3)$), $m$ is the quark mass and $M$ is the gluon mass. The above Lagrangian is constrained by the Lorentz condition

$$\partial^{\mu}A_{\mu}^{a}=0. \tag{2.2}$$

Under this condition, as was proved in Refs. [6-8], the action given by the Lagrangian in Eq. (2.1) is invariant with respect to the following gauge transformations:

$$\begin{gathered}\delta A_{\mu}^{a}=\xi D_{\mu}^{ab}C^{b},\\ \delta\psi(x)=ig\xi T^{a}C^{a}(x)\psi(x),\\ \delta\bar{\psi}(x)=ig\xi\bar{\psi}(x)T^{a}C^{a}(x)\end{gathered} \tag{2.3}$$

where

$$D_{\mu}^{ab}=\delta^{ab}\partial_{\mu}-gf^{abc}A_{\mu}^{c} \tag{2.4}$$

is the covariant derivative. In the above, we have set the parametric functions of the gauge group $\theta^a(x)=\xi C^a(x)$ in which $\xi$ is an infinitesimal Grassmann number and $C^a(x)$ are the ghost field functions.

The quantization of the massive QCD was carried out by different approaches in Ref. [6]. A simpler quantization is performed in the Lagrangian path-integral formalism by means of the Lagrange undetermined multiplier method which was shown to be equivalent to the Faddeev-Popov approach of quantization [9]. For this quantization, it is convenient to generalize the QCD Lagrangian and the Lorentz condition to the following forms:

$$\mathcal{L}_\lambda = \bar{\psi}\{i\gamma^\mu(\partial_\mu - igT^aA^a_\mu) - m\}\psi - \frac{1}{4}F^{a\mu\nu}F^a_{\mu\nu} + \frac{1}{2}M^2A^{a\mu}A^a_\mu - \frac{1}{2}\alpha(\lambda^a)^2 \tag{2.5}$$

and

$$\partial^\mu A^a_\mu + \alpha\lambda^a = 0 \tag{2.6}$$

where $\lambda^a(x)$ are the extra functions which will be identified with the Lagrange multipliers and $\alpha$ is an arbitrary constant playing the role of gauge parameter. According to the general procedure for constrained systems, the constraint in Eq. (2.6) may be incorporated into the Lagrangian in Eq. (2.5) by the Lagrange multiplier method, giving a generalized Lagrangian such that

$$\mathcal{L}_\lambda = \bar{\psi}\{i\gamma^\mu(\partial_\mu - igT^aA^a_\mu) - m\}\psi - \frac{1}{4}F^{a\mu\nu}F^a_{\mu\nu} + \frac{1}{2}M^2A^{a\mu}A^a_\mu + \lambda^a\partial^\mu A^a_\mu + \frac{1}{2}\alpha(\lambda^a)^2. \tag{2.7}$$

This Lagrangian is obviously not gauge-invariant. However, for building up a correct gauge field theory, it is necessary to require the dynamics of the system, i.e. the action given by the Lagrangian (2.7) to be invariant under the gauge transformations denoted in Eq. (2.3). By this requirement, noticing the identity $f^{abc}A^{a\mu}A^b_\mu = 0$ and applying the constraint condition in Eq.(2.6), we find

$$\delta S_\lambda = -\frac{\xi}{\alpha}\int d^4x\partial^\nu A^a_\nu(x)\partial^\mu(\mathcal{D}^{ab}_\mu(x)C^b(x)) = 0 \tag{2.8}$$

where

$$\mathcal{D}^{ab}_\mu(x) = \delta^{ab}\frac{\sigma^2}{\Box_x}\partial^x_\mu + D^{ab}_\mu(x) \tag{2.9}$$

in which $\sigma^2 = \alpha M^2$, $\Box_x$ is the D'Alembertian operator and $D^{ab}_\mu(x)$ was defined in Eq. (2.4). From Eq. (2.6) we see $\frac{1}{\alpha}\partial^\nu A^a_\nu = -\lambda^a \neq 0$. Therefore, to ensure the action to be gauge-invariant, the following constraint condition on the gauge group is necessary to be required

$$\partial^\mu_x(\mathcal{D}^{ab}_\mu(x)C^b(x)) = 0 \tag{2.10}$$

which usually is called ghost equation. When this constraint condition is incorporated into the Lagrangian in Eq. (2.7) by the Lagrange multiplier method, we obtain a more generalized Lagrangian as follows

$$\mathcal{L}_\lambda = \bar{\psi}\{i\gamma^\mu(\partial_\mu - igT^aA^a_\mu) - m\}\psi - \frac{1}{4}F^{a\mu\nu}F^a_{\mu\nu} + \frac{1}{2}M^2A^{a\mu}A^a_\mu + \lambda^a\partial^\mu A^a_\mu + \frac{1}{2}\alpha(\lambda^a)^2 + \bar{C}^a\partial^\mu(\mathcal{D}^{ab}_\mu C^b) \tag{2.11}$$

where $\bar{C}^a(x)$, acting as Lagrange undetermined multipliers, are the new scalar variables conjugate to the ghost variables $C^a(x)$.

As we learn from the Lagrange undetermined multiplier method, the dynamical and constrained variables as well as the Lagrange multipliers in the Lagrangian (2.11) can all be treated as free ones, varying arbitrarily. Therefore, we are allowed to use this kind of Lagrangian to construct the generating functional of Green functions

$$\begin{aligned} Z[J^{a\mu},\overline{\eta},\eta,\overline{\xi}^a,\xi^a] &= \frac{1}{N}\int D(A^a_\mu,\bar{\psi},\psi,\bar{C}^a,C^a,\lambda^a)exp\{i\int d^4x[\mathcal{L}_\lambda(x) \\ &+ J^{a\mu}(x)A^a_\mu(x) + \bar{\psi}\eta + \overline{\eta}\psi + \overline{\xi}^a(x)C^a(x) + \bar{C}^a(x)\xi^a(x)]\} \\ &= \frac{1}{N}\int D(A^a_\mu,\bar{\psi},\psi,\bar{C}^a,C^a,)exp\{iS + \int d^4x[J^{a\mu}(x)A^a_\mu(x) \\ &+ \bar{\psi}\eta + \overline{\eta}\psi + \overline{\xi}^a(x)C^a(x) + \bar{C}^a(x)\xi^a(x)]\} \end{aligned} \tag{2.12}$$

where the last equality is obtained by carrying out the integral over $\lambda^a(x)$, $D(A^a_\mu,\cdots,\lambda^a)$ denotes the functional integration measure, $J^a_\mu,\overline{\eta},\eta,\overline{\xi}^a$ and $\xi^a$ are the external sources coupled to the gluon, quark and ghost fields, $N$ is the normalization constant and

$$S = \int d^4x[\bar{\psi}\{i\gamma^\mu(\partial_\mu - igT^aA^a_\mu) - m\}\psi - \frac{1}{4}F^{a\mu\nu}F^a_{\mu\nu} + \frac{1}{2}M^2A^{a\mu}A^a_\mu - \frac{1}{2\alpha}(\partial^\mu A^a_\mu)^2 - \partial^\mu\bar{C}^a\mathcal{D}^{ab}_\mu C^b] \tag{2.13}$$

is the effective action given in arbitrary gauges.

Similar to the massless QCD, for the massive QCD, there are a set of BRST-transformations including the infinitesimal gauge transformations shown in Eq. (2.3) and the transformations for the ghost fields under which the effective action is invariant. The transformations for the ghost fields may be found from the stationary condition of the effective action under the BRST-transformations. By applying the transformations in Eq. (2.3) to the action in Eq. (2.13), one can derive

$$\delta S = \int d^4x \{[\delta \bar{C}^a - \frac{\xi}{\alpha}\partial^\nu A_\nu^a]\partial^\mu(\mathcal{D}_\mu^{ab}C^b) + \bar{C}^a \partial^\mu \delta(\mathcal{D}_\mu^{ab}C^b)\} = 0. \tag{2.14}$$

This expression suggests that if we set

$$\delta \bar{C}^a = \frac{\xi}{\alpha}\partial^\nu A_\nu^a \tag{2.15}$$

and

$$\partial^\mu \delta(\mathcal{D}_\mu^{ab}C^b) = 0. \tag{2.16}$$

The action will be invariant. Eq. (2.15) gives the transformation law of the ghost field variable $\bar{C}^a(x)$ which is the same as the one in the massless gauge field theory. From Eq. (2.16), we may derive a transformation law of the ghost field variables $C^a(x)$. Noticing the relation in Eq. (2.9), we can write

$$\delta(\mathcal{D}_\mu^{ab}(x)C^b(x)) = \frac{\sigma^2}{\Box_x}\partial_\mu^x \delta C^a(x) + \delta(D_\mu^{ab}(x)C^b(x)). \tag{2.17}$$

In the massless QCD, it has been proved that [13-17]

$$\delta(D_\mu^{ab}(x)C^b(x)) = D_\mu^{ab}(x)[\delta C^b(x) + \frac{\xi}{2} g f^{bcd} C^c(x) C^d(x)]. \tag{2.18}$$

With this result, Eq. (2.17) can be written as

$$\delta(\mathcal{D}_\mu^{ab}(x)C^b(x)) = \mathcal{D}_\mu^{ab}(x)\delta C^b(x) - D_\mu^{ab}(x)\delta C_0^b(x) \tag{2.19}$$

where

$$\delta C_0^a(x) \equiv -\frac{\xi g}{2} f^{abc} C^b(x) C^c(x). \tag{2.20}$$

On substituting Eq. (2.19) into Eq. (2.16), we have

$$M^{ab}(x)\delta C^b(x) = M_0^{ab}(x)\delta C_0^b(x) \tag{2.21}$$

where we have defined

$$M^{ab}(x) \equiv \partial_x^\mu \mathcal{D}_\mu^{ab}(x) = \delta^{ab}(\Box_x + \sigma^2) - g f^{abc} A_\mu^c(x)\partial_x^\mu \tag{2.22}$$

and

$$M_0^{ab}(x) \equiv \partial_x^\mu D_\mu^{ab}(x) = M^{ab}(x) - \sigma^2 \delta^{ab}. \tag{2.23}$$

It is noted that the operator in Eq. (2.22 ) is just the operator appearing in Eq. (2.10). Corresponding to Eq.(2.10), we may write an equation satisfied by the Green function $\Delta^{ab}(x-y)$

$$M^{ac}(x)\Delta^{cb}(x-y) = \delta^{ab}\delta^4(x-y). \tag{2.24}$$

The function $\Delta^{ab}(x-y)$ is nothing but the exact propagator of the ghost field which is the inverse of the operator $M^{ab}(x)$. In the light of Eq. (2.24) and noticing Eq. (2.23), we may solve out the $\delta C^a(x)$ from Eq. (2.21)

$$\begin{aligned} \delta C^a(x) &= (M^{-1}M_0\delta C_0)^a(x) = \{M^{-1}(M-\sigma^2)\delta C_0\}^a(x) \\ &= \delta C_0^a(x) - \sigma^2 \int d^4y \Delta^{ab}(x-y)\delta C_0^b(y). \end{aligned} \tag{2.25}$$

This just is the transformation law for the ghost field variables $C^a(x)$. When the gluon mass $M$ tends to zero, Eq. (2.25) immediately goes over to the corresponding transformation given in the massless gauge field theory. It is interesting that in the Landau gauge ($\alpha = 0$), due to $\sigma = 0$, the above transformation also reduces to the form as given in the massless theory. This result is natural since in the Landau gauge, the gluon field mass term in the action is gauge-invariant. However, in general gauges, the mass term is no longer gauge-invariant. In this case, to maintain the action to be gauge-invariant, it is necessary to give the ghost field a mass $\sigma$ so as to counteract the gauge-non-invariance of the gluon field mass term. As a result, in the transformation given in Eq. (2.25) appears a term proportional to $\sigma^2$.

## III. WARD-TAKAHASHI IDENTITIES

This section is devoted to deriving the W-T identities for massive QCD on the basis of the BRST-symmetry of the theory. Since the derivations are much similar to those for the QCD with massless gluons, we only need here to give a brief description of the derivations. When we make the BRST-transformations shown in Eqs. (2.3), (2.15) and (2.25) to the generating functional in Eq. (2.12) and consider the invariance of the generating functional, the action and the integration measure under the transformations (the invariance of the integration measure is easy to check), we obtain an identity such that [11-17]

$$\begin{array}{c}\frac{1}{N}\int\mathcal{D}(A_\mu^a,\bar{C}^a,C^a,\bar{\psi},\psi)\int d^4x\{J^{a\mu}(x)\delta A_\mu^a(x)+\delta\overline{C}^a(x)\xi^a(x)+\overline{\xi}^a(x)\delta C^a(x)\\ +\bar{\eta}(x)\delta\psi(x)+\delta\bar{\psi}(x)\eta(x)\}e^{iS+EST}\\ =0\end{array}\tag{3.1}$$

where $EST$ is an abbreviation of the external source terms appearing in Eq. (2.12). The Grassmann number $\xi$ contained in the BRST-transformations in Eq. (3.1) may be eliminated by performing a partial differentiation of Eq. (3.1) with respect to $\xi$. As a result, we get a W-T identity as follows

$$\begin{array}{c}\frac{1}{N}\int\mathcal{D}(A_\mu^a,\bar{C}^a,C^a,\bar{\psi},\psi)\int d^4x\{J^{a\mu}(x)\Delta A_\mu^a(x)+\triangle\overline{C}^a(x)\xi^a(x)-\overline{\xi}^a(x)\Delta C^a(x)\\ -\bar{\eta}(x)\Delta\psi(x)+\Delta\bar{\psi}(x)\eta(x)\}e^{iS+EST}\\ =0\end{array}\tag{3.2}$$

where

$$\begin{array}{c}\Delta A_\mu^a(x)=D_\mu^{ab}(x)C^b(x),\\ \Delta\bar{C}^a(x)=\frac{1}{\alpha}\partial^\mu A_\mu^a(x),\\ \Delta C^a(x)=\int d^4y[\delta^{ab}\delta^4(x-y)-\sigma^2\Delta^{ab}(x-y)]\triangle C_0^b(y),\\ \Delta C_0^b(y)=-\frac{1}{2}gf^{bcd}C^c(y)C^d(y),\\ \Delta\psi(x)=igT^aC^a(x)\psi(x),\\ \Delta\bar{\psi}(x)=ig\bar{\psi}(x)T^aC^a(x).\end{array}\tag{3.3}$$

These functions defined above are finite. Each of them differs from the corresponding BRST-transformation written in Eqs. (2.3), (2.15) and (2.25) by an infinitesimal Grassmann parameter $\xi$.

In order to represent the composite field functions $\Delta A_\mu^a,\Delta C^a,\Delta\bar{\psi}$ and $\Delta\psi$ in Eq. (3.2) in terms of differentials of the functional $Z$ with respect to external sources, we may, as usual, construct a generalized generating functional by introducing new external sources (called BRST-sources later on) into the generating functional written in Eq. (2.12), as shown in the following [13-16]

$$\begin{array}{c}Z[J_\mu^a,\overline{\xi}^a,\xi^a,\bar{\eta},\eta;u^{a\mu},v^a,\bar{\zeta},\zeta]\\ =\frac{1}{N}\int\mathcal{D}[A_\mu^a,\bar{C}^a,C^a,\bar{\psi},\psi]exp\{iS+i\int d^4x[u^{a\mu}\Delta A_\mu^a+v^a\Delta C^a\\ +\Delta\bar{\psi}\zeta+\bar{\zeta}\Delta\psi+J^{a\mu}A_\mu^a+\overline{\xi}^aC^a+\bar{C}^a\xi^a+\bar{\eta}\psi+\bar{\psi}\eta]\}\end{array}\tag{3.4}$$

where $u^{a\mu}$, $v^a$, $\overline{\varsigma}$ and $\varsigma$ are the sources which belong to the corresponding functions $\Delta A_\mu^a$, $\Delta C^a$, $\Delta\Psi$ and $\Delta\overline{\psi}$, respectively. Obviously, the $u^{a\mu}$ and $\Delta A_\mu^a$ are anticommuting quantities, while, the $v^a$, $\bar{\zeta}$, $\zeta$, $\Delta C^a$, $\Delta\bar{\psi}$ and $\Delta\psi$ are commuting ones. We may start from the above generating functional to re-derive the W-T identity. In order that the identity thus derived is identical to that as given in Eq. (3.2), it is necessary to require the BRST-source terms $u_i\Delta\Phi_i$, where $u_i=u^{a\mu}$, $v^a$, $\overline{\zeta}$ or $\zeta$ and $\Delta\Phi_i=\Delta A_\mu^a$, $\Delta C^a$, $\Delta\Psi$ or $\Delta\overline{\Psi}$ to be invariant under the BRST-transformations. How to ensure the BRST-invariance of the source terms? For illustration, let us introduce the source terms in such a fashion

$$\begin{array}{c}\int d^4x[\widetilde{u}^{a\mu}\delta A_\mu^a+\widetilde{v}^a\delta C^a+\overline{\widetilde{\zeta}}\delta\psi+\delta\overline{\psi}\widetilde{\zeta}]\\ =\int d^4x[u^{a\mu}\triangle A_\mu^a+v^a\triangle C^a+\overline{\zeta}\triangle\psi+\triangle\overline{\psi}\zeta]\end{array}\tag{3.5}$$

where

$$u^{a\mu}=\tilde{u}^{a\mu}\xi,\ \ v^a=\tilde{v}^a\xi,\ \ \bar{\varsigma}=\overline{\tilde{\varsigma}}\xi,\ \ \varsigma=-\tilde{\varsigma}\xi.\tag{3.6}$$

These external sources are defined by including the Grassmann number $\xi$ and hence products of them with $\xi$ vanish. This suggests that we may generally define the sources by the following condition

$$u_i\xi = 0. \tag{3.7}$$

Considering that under the BRST-transformation, the variation of the composite field functions given in the general gauges can be represented in the form $\delta\Delta\Phi_i = \xi\tilde{\Phi}_i$ where $\tilde{\Phi}_i$ are functions without including the parameter $\xi$, clearly, the definition in Eq. (3.7) for the sources would guarantee the BRST- invariance of the BRST-source terms. When the BRST-transformations in Eqs. (2.3), (2.15) and (2.25) are made to the generating functional in Eq. (3.4), due to the definition in Eq. (3.7) for the sources, we have $u_i\delta\Delta\Phi_i = 0$ which means that the BRST-source terms give a vanishing contribution to the identity in Eq. (3.1). Therefore, we still obtain the identity as shown in Eq. (3.1) except that the external source terms is now extended to include the BRST-external source terms. This fact indicates that we may directly insert the BRST-source terms into the exponent in Eq. (3.1) without changing the identity itself. When performing a partial differentiation of the identity with respect to $\xi$, we obtain a W-T identity which is the same as written in Eq. (3.2) except that the BRST-source terms are now included in the identity. Therefore, Eq. (3.2) may be expressed as

$$\begin{array}{c}\int d^4x[J^{a\mu}(x)\frac{\delta}{\delta u^{a\mu}(x)} - \overline{\xi}^a(x)\frac{\delta}{\delta v^a(x)} - \bar{\eta}(x)\frac{\delta}{\delta\bar{\zeta}(x)} \\ +\eta(x)\frac{\delta}{\delta\zeta(x)} + \frac{1}{\alpha}\xi^a(x)\partial_x^\mu\frac{\delta}{\delta J^{a\mu}(x)}]Z[J_\mu^a,\cdots,\zeta] \\ = 0.\end{array} \tag{3.8}$$

This is the W-T identity satisfied by the generating functional of full Green functions.

On substituting in Eq. (3.8) the relation [13-16]

$$Z = e^{iW} \tag{3.9}$$

where $W$ denotes the generating functional of connected Green functions, one may obtain a W-T identity expressed by the functional $W$

$$\begin{array}{c}\int d^4x[J^{a\mu}(x)\frac{\delta}{\delta u^{a\mu}(x)} - \overline{\xi}^a(x)\frac{\delta}{\delta v^a(x)} - \bar{\eta}(x)\frac{\delta}{\delta\bar{\zeta}(x)} + \eta(x)\frac{\delta}{\delta\zeta(x)} \\ +\frac{1}{\alpha}\xi^a(x)\partial_x^\mu\frac{\delta}{\delta J^{a\mu}(x)}]W[J_u^a,\cdots,\zeta] \\ = 0.\end{array} \tag{3.10}$$

From this identity, one may get another W-T identity satisfied by the generating functional $\Gamma$ of proper (one-particle-irreducible) vertex functions. The functional $\Gamma$ is usually defined by the following Legendre transformation [13-16]

$$\begin{array}{c}\Gamma[A^{a\mu},\bar{C}^a,C^a,\bar{\psi},\psi;u_\mu^a,v^a,\bar{\zeta},\zeta] = W[J_\mu^a,\overline{\xi}^a,\xi^a,\bar{\eta},\eta;u_\mu^a,v^a,\bar{\zeta},\zeta] \\ -\int d^4x[J_\mu^aA^{a\mu} + \overline{\xi}^aC^a + \bar{C}^a\xi^a + \bar{\eta}\psi + \bar{\psi}\eta]\end{array} \tag{3.11}$$

where $A_\mu^a,\bar{C}^a,C^a,\bar{\psi}$ and $\psi$ are the field variables defined by the following functional derivatives

$$\begin{array}{c}A_\mu^a(x) = \frac{\delta W}{\delta J^{a\mu}(x)},\ \ \bar{C}^a(x) = -\frac{\delta W}{\delta\xi^a(x)}, C^a(x) = \frac{\delta W}{\delta\overline{\xi}^a(x)}, \\ \bar{\psi}(x) = -\frac{\delta W}{\delta\eta(x)},\ \ \psi(x) = \frac{\delta W}{\delta\bar{\eta}(x)}.\end{array} \tag{3.12}$$

From Eq.(3.11), it is not difficult to get the inverse transformations [13-16]

$$\begin{array}{c}J^{a\mu}(x) = -\frac{\delta\Gamma}{\delta A_\mu^a(x)},\ \ \overline{\xi}^a(x) = \frac{\delta\Gamma}{\delta C^a(x)}, \xi^a(x) = -\frac{\delta\Gamma}{\delta\bar{C}^a(x)}, \\ \bar{\eta}(x) = \frac{\delta\Gamma}{\delta\psi(x)},\ \ \eta(x) = -\frac{\delta\Gamma}{\delta\bar{\psi}(x)}.\end{array} \tag{3.13}$$

It is obvious that

$$\frac{\delta W}{\delta u_\mu^a} = \frac{\delta\Gamma}{\delta u_\mu^a},\ \ \frac{\delta W}{\delta v^a} = \frac{\delta\Gamma}{\delta v^a},\ \ \frac{\delta W}{\delta\zeta} = \frac{\delta\Gamma}{\delta\zeta},\ \ \frac{\delta W}{\delta\bar{\zeta}} = \frac{\delta\Gamma}{\delta\bar{\zeta}}. \tag{3.14}$$

Employing Eqs. (3.13) and (3.14), the W-T identity in Eq. (3.10) will be written as [13-16]

$$\begin{array}{c}\int d^4x\{\frac{\delta\Gamma}{\delta A_\mu^a(x)}\frac{\delta\Gamma}{\delta u^{a\mu}(x)} + \frac{\delta\Gamma}{\delta C^a(x)}\frac{\delta\Gamma}{\delta v^a(x)} + \frac{\delta\Gamma}{\delta\psi(x)}\frac{\delta\Gamma}{\delta\bar{\zeta}(x)} \\ +\frac{\delta\Gamma}{\delta\bar{\psi}(x)}\frac{\delta\Gamma}{\delta\zeta(x)} + \frac{1}{\alpha}\partial_x^\mu A_\mu^a(x)\frac{\delta\Gamma}{\delta\overline{C}^a(x)}\} \\ = 0.\end{array} \tag{3.15}$$

This is the W-T identity satisfied by the generating functional of proper vertex functions.

The above identity may be represented in another form with the aid of the so-called ghost equation of motion. The ghost equation may easily be derived by firstly making the translation transformation: $\bar{C}^a \rightarrow \bar{C}^a + \bar{\lambda}^a$ in Eq. (2.12) where $\bar{\lambda}^a$ is an arbitrary Grassmann variable, then differentiating Eq. (2.12) with respect to the $\bar{\lambda}^a$ and finally setting $\overline{\lambda}^a = 0$. The result is [13-16]

$$\frac{1}{N}\int D(A_\mu^a, \bar{C}^a, C^a, \bar{\psi}, \psi)\{\xi^a(x) + \partial_x^\mu(\mathcal{D}_\mu^{ab}(x)C^b(x))\}e^{iS+EST} = 0. \tag{3.16}$$

When we use the generating functional defined in Eq. (3.4) and notice the relation in Eq. (2.9), the above equation may be represented as [13-16]

$$[\xi^a(x) - i\partial_x^\mu \frac{\delta}{\delta u^{a\mu}(x)} - i\sigma^2 \frac{\delta}{\delta \overline{\xi}^a(x)}]Z[J_\mu^a, \cdots, \zeta] = 0. \tag{3.17}$$

On substituting the relation in Eq. (3.9) into the above equation, we may write a ghost equation satisfied by the functional $W$ such that

$$\xi^a(x) + \partial_x^\mu \frac{\delta W}{\delta u^{a\mu}(x)} + \sigma^2 \frac{\delta W}{\delta \overline{\xi}^a(x)} = 0. \tag{3.18}$$

From this equation, the ghost equation obeyed by the functional $\Gamma$ is easy to be derived by virtue of Eqs. (3.12) - (3.14) [13-16]

$$\frac{\delta \Gamma}{\delta \bar{C}^a(x)} - \partial_x^\mu \frac{\delta \Gamma}{\delta u^{a\mu}(x)} - \sigma^2 C^a(x) = 0. \tag{3.19}$$

Upon applying the above equation to the last term in Eq. (3.15). the identity in Eq. (3.15) will be rewritten as

$$\begin{array}{c} \int d^4x\{\frac{\delta\Gamma}{\delta A_\mu^a}\frac{\delta\Gamma}{\delta u^{a\mu}} + \frac{\delta\Gamma}{\delta C^a}\frac{\delta\Gamma}{\delta v^a} + \frac{\delta\Gamma}{\delta\psi}\frac{\delta\Gamma}{\delta\bar{\zeta}} + \frac{\delta\Gamma}{\delta\bar{\psi}}\frac{\delta\Gamma}{\delta\zeta} \\ +M^2\partial^\nu A_\nu^a C^a - \frac{1}{\alpha}\partial^\mu\partial^\nu A_\nu^a \frac{\delta\Gamma}{\delta u^{a\mu}}\} \\ = 0. \end{array} \tag{3.20}$$

Now, let us define a new functional $\hat{\Gamma}$ in such a manner

$$\hat{\Gamma} = \Gamma + \frac{1}{2\alpha}\int d^4x(\partial^\mu A_\mu^a)^2. \tag{3.21}$$

From this definition, it follows that

$$\frac{\delta\Gamma}{\delta A_\mu^a} = \frac{\delta\hat{\Gamma}}{\delta A_\mu^a} + \frac{1}{\alpha}\partial^\mu\partial^\nu A_\nu^a. \tag{3.22}$$

When inserting Eq. (3.21) into Eq. (3.20) and considering the relation in Eq. (3.22), we arrive at

$$\int d^4x\{\frac{\delta\hat{\Gamma}}{\delta A_\mu^a}\frac{\delta\hat{\Gamma}}{\delta u^{a\mu}} + \frac{\delta\hat{\Gamma}}{\delta C^a}\frac{\delta\hat{\Gamma}}{\delta v^a} + \frac{\delta\hat{\Gamma}}{\delta\psi}\frac{\delta\hat{\Gamma}}{\delta\bar{\zeta}} + \frac{\delta\hat{\Gamma}}{\delta\bar{\psi}}\frac{\delta\hat{\Gamma}}{\delta\zeta} + M^2\partial^\nu A_\nu^a C^a\} = 0. \tag{3.23}$$

The ghost equation represented through the functional $\hat{\Gamma}$ is of the same form as Eq. (3.19)

$$\frac{\delta\hat{\Gamma}}{\delta\bar{C}^a(x)} - \partial_x^\mu\frac{\delta\hat{\Gamma}}{\delta u^{a\mu}(x)} - \sigma^2 C^a(x) = 0. \tag{3.24}$$

In the Landau gauge, since $\sigma = 0$ and $\partial^\nu A_\nu^a = 0$, Eqs. (3.23) and (3.24) respectively reduce to [13-16]

$$\int d^4x\{\frac{\delta\hat{\Gamma}}{\delta A_\mu^a}\frac{\delta\hat{\Gamma}}{\delta u^{a\mu}} + \frac{\delta\hat{\Gamma}}{\delta C^a}\frac{\delta\hat{\Gamma}}{\delta v^a} + \frac{\delta\hat{\Gamma}}{\delta\psi}\frac{\delta\hat{\Gamma}}{\delta\bar{\zeta}} + \frac{\delta\hat{\Gamma}}{\delta\bar{\psi}}\frac{\delta\hat{\Gamma}}{\delta\zeta}\} = 0 \tag{3.25}$$

and

$$\frac{\delta\hat{\Gamma}}{\delta\bar{C}^a} - \partial^\mu\frac{\delta\hat{\Gamma}}{\delta u^{a\mu}} = 0. \tag{3.26}$$

These equations formally are the same as those for the massless QCD.

From the W-T identities formulated above, we may derive various W-T identities obeyed by Green functions and vertices, as will be illustrated in the next sections.

## IV. GLUON AND GHOST PARTICLE PROPAGATORS

In this section, we plan to derive the W-T identities satisfied by the massive gluon and ghost particle propagators by starting from the W-T identity represented in Eq. (3.8) and the ghost equation shown in Eq. (3.17) and then discuss their renormalization. Let us perform differentiations of the identities in Eqs. (3.8) and (3.17) with respect to the external sources $\xi^a(x)$ and $\xi^b(y)$ respectively and then set all the sources except for the source $J^a_\mu(x)$ to be zero. In this way, we obtain the following identities

$$\frac{1}{\alpha}\partial^\mu_x \frac{\delta Z[J]}{\delta J^{a\mu}(x)} + \int d^4y J^{b\nu}(y)\frac{\delta^2 Z[J,\xi,u]}{\delta\xi^a(x)\delta u^{b\nu}(y)}|_{\xi=u=0} = 0 \tag{4.1}$$

and

$$\begin{array}{c} i\partial^x_\mu \frac{\delta^2 Z[J.\xi.u]}{\delta u^a_\mu(x)\delta\xi^b(y)}|_{\xi=u=0} + i\sigma^2 \frac{\delta^2 Z[J,\overline{\xi},\xi]}{\delta\overline{\xi}^a(x)\delta\xi^b(y)}|_{\overline{\xi}=\xi=0} \\ +\delta^{ab}\delta^4(x-y)Z[J] = 0. \end{array} \tag{4.2}$$

Furthermore, on differentiating Eq. (4.1) with respect to $J^b_\nu(y)$ and then letting the source $J$ vanish, we may get an identity which is, in operator representation, of the form [13-16]

$$\frac{1}{\alpha}\partial^\mu_x < 0^+|T[\hat{A}^a_\mu(x)\hat{A}^b_\nu(y)]|0^- >=< 0^+|T^*[\hat{\bar{C}}^a(x)\hat{D}^{bd}_\nu(y)\hat{C}^d(y)]|0^- > \tag{4.3}$$

where $\hat{A}^a_\nu(x)$, $\hat{C}^a(x)$ and $\hat{\bar{C}}^a(x)$ stand for the gluon field and ghost field operators and $T^*$ symbolizes the covariant time-ordering product. When the source $J$ is set to vanish, Eq. (4.2) gives such an equation [13-16]

$$\begin{array}{c} i\partial^\nu_y < 0^+|T^*\{\hat{\bar{C}}^a(x)\hat{D}^{bd}_\nu(y)\hat{C}^d(y)\}|0^- > \\ +i\sigma^2 < 0^+|T[\hat{\bar{C}}^a(x)\hat{C}^b(y)]|0^- >= \delta^{ab}\delta^4(x-y). \end{array} \tag{4.4}$$

Upon inserting Eq. (4.3) into Eq. (4.4), we have

$$\partial^\mu_x\partial^\nu_y D^{ab}_{\mu\nu}(x-y) - \alpha\sigma^2\Delta^{ab}(x-y) = -\alpha\delta^{ab}\delta^4(x-y) \tag{4.5}$$

where

$$iD^{ab}_{\mu\nu}(x-y) =< 0^+|T\{\hat{A}^a_\mu(x)\hat{A}^b_\nu(y)\}|0^- > \tag{4.6}$$

which is the familiar full gluon propagator and

$$i\Delta^{ab}(x-y) =< 0^+|T\{\hat{C}^a(x)\hat{\bar{C}}^b(y)\}|0^- > \tag{4.7}$$

which is the full ghost particle propagator. Eq. (4.5) just is the W-T identity respected by the gluon propagator which establishes a relation between the longitudinal part of gluon propagator and the ghost particle propagator. Particularly, in the Landau gauge ($\alpha = 0$), as we see, Eq. (4.5) reduces to the form which exhibits the transversity of the gluon propagator. By the Fourier transformation, Eq. (4.5) will be converted to the form given in the momentum space as follows

$$k^\mu k^\nu D^{ab}_{\mu\nu}(k) - \alpha\sigma^2\Delta^{ab}(k) = -\alpha\delta^{ab}. \tag{4.8}$$

The ghost particle propagator may be determined by the ghost equation shown in Eq. (4.4). However, we would rather here to derive its expression from the Dyson-Schwinger equation [29] satisfied by the propagator which may be established by the perturbation method.

$$\Delta^{ab}(k) = \Delta^{ab}_0(k) + \Delta^{aa'}_0(k)\Omega^{a'b'}(k)\Delta^{b'b}(k) \tag{4.9}$$

where

$$i\Delta^{ab}_0(k) = i\delta^{ab}\Delta_0(k) = \frac{-i\delta^{ab}}{k^2-\sigma^2+i\varepsilon} \tag{4.10}$$

is the free ghost particle propagator which can be derived from the generating functional in Eq. (2.12) by a perturbative calculation and $-i\Omega^{ab}(k) = -i\delta^{ab}\Omega(k)$ denotes the proper self-energy operator of ghost particle. From Eq. (4.9), it is easy to solve that

$$i\Delta^{ab}(k) = \frac{-i\delta^{ab}}{k^2[1+\hat{\Omega}(k^2)] - \sigma^2 + i\varepsilon} \tag{4.11}$$

where the self-energy has properly been expressed as

$$\Omega(k) = k^2\hat{\Omega}(k^2). \tag{4.12}$$

Similarly, we may write a Dyson-Schwinger equation for the gluon propagator by the perturbation procedure [29]

$$D_{\mu\nu}(k) = D^0_{\mu\nu}(k) + D^0_{\mu\lambda}(k)\Pi^{\lambda\rho}(k)D_{\rho\nu}(k) \tag{4.13}$$

where the color indices are suppressed for simplicity and

$$iD^{(0)ab}_{\mu\nu}(k) = i\delta^{ab}D^{(0)}_{\mu\nu}(k) = -i\delta^{ab}[\frac{g_{\mu\nu} - k_\mu k_\nu/k^2}{k^2 - M^2 + i\varepsilon} + \frac{\alpha k_\mu k_\nu/k^2}{k^2 - \sigma^2 + i\varepsilon}] \tag{4.14}$$

is the free gluon propagator which can easily be derived from the perturbative expansion of the generating functional in Eq. (2.12) and $-i\Pi^{ab}_{\mu\nu}(k) = -i\delta^{ab}\Pi_{\mu\nu}(k)$ stands for the gluon proper self-energy operator. Let us decompose the propagator and the self-energy operator into a transverse part and a longitudinal part:

$$D^{\mu\nu}(k) = D^{\mu\nu}_T(k) + D^{\mu\nu}_L(k), \Pi^{\mu\nu}(k) = \Pi^{\mu\nu}_T(k) + \Pi^{\mu\nu}_L(k) \tag{4.15}$$

where

$$\begin{aligned} D^{\mu\nu}_T(k) = \mathcal{P}^{\mu\nu}_T(k)D_T(k^2),\ D^{\mu\nu}_L(k) = \mathcal{P}^{\mu\nu}_L(k)D_L(k^2), \\ \Pi^{\mu\nu}_T(k) = \mathcal{P}^{\mu\nu}_T(k)\Pi_T(k^2),\ \Pi^{\mu\nu}_L(k) = \mathcal{P}^{\mu\nu}_L(k)\Pi_L(k^2) \end{aligned} \tag{4.16}$$

here $\mathcal{P}^{\mu\nu}_T(k) = (g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2})$ and $\mathcal{P}^{\mu\nu}_L(k) = \frac{k^\mu k^\nu}{k^2}$ are the transverse and longitudinal projectors respectively. Considering these decompositions and the orthogonality between the transverse and longitudinal parts, Eq. (4.13) will be split into two equations

$$D_{T\mu\nu}(k) = D^0_{T\mu\nu}(k) + D^0_{T\mu\lambda}(k)\Pi^{\lambda\rho}_T(k)D_{T\rho\nu}(k) \tag{4.17}$$

and

$$D_{L\mu\nu}(k) = D^0_{L\mu\nu}(k) + D^0_{L\mu\lambda}(k)\Pi^{\lambda\rho}_L(k)D_{L\rho\nu}(k). \tag{4.18}$$

Solving the equations (4.17) and (4.18), one can get

$$iD^{ab}_{\mu\nu}(k) = -i\delta^{ab}\{\frac{g_{\mu\nu} - k_\mu k_\nu/k^2}{k^2 + \Pi_T(k^2) - M^2 + i\varepsilon} + \frac{\alpha k_\mu k_\nu/k^2}{k^2 + \alpha\Pi_L(k^2) - \sigma^2 + i\varepsilon}\}. \tag{4.19}$$

With setting

$$\Pi_T(k^2) = k^2\Pi_1(k^2) + M^2\Pi_2(k^2) \tag{4.20}$$

which follows from the Lorentz-covariance of the operator $\Pi_T(k^2)$ and

$$\alpha\Pi_L(k^2) = k^2\hat{\Pi}_L(k^2), \tag{4.21}$$

Eq. (4.19) will be written as

$$iD^{ab}_{\mu\nu}(k) = -i\delta^{ab}\{\frac{g_{\mu\nu} - k_\mu k_\nu/k^2}{k^2[1+\Pi_1(k^2)] - M^2[1-\Pi_2(k^2)] + i\varepsilon} + \frac{\alpha k_\mu k_\nu/k^2}{k^2[1+\hat{\Pi}_L(k^2)] - \sigma^2 + i\varepsilon}. \tag{4.22}$$

We would like to note that the expressions given in Eqs. (4.12), (4.20) and (4.21) can be verified by practical calculations and are important for renormalizations of the propagators and the gluon mass.

Substitution of Eqs. (4.11) and (4.22) into Eq. (4.8) yields

$$\hat{\Pi}_L(k^2) = \frac{\sigma^2\hat{\Omega}(k^2)}{k^2[1+\hat{\Omega}(k^2)]}. \tag{4.23}$$

From this relation, we see, either in the Landau gauge or in the zero-mass limit, the $\hat{\Pi}_L(k^2)$ vanishes.

Now let us discuss the renormalization. The function $\hat{\Omega}(k^2)$ in Eq. (4.11) and the functions $\Pi_1(k^2)$, $\Pi_2(k^2)$ and $\hat{\Pi}_L(k^2)$ in Eq. (4.22) are generally divergent in higher order perturbative calculations. According to the conventional procedure of renormalization, the divergences included in the functions $\hat{\Omega}(k^2)$, $\Pi_1(k^2)$, $\Pi_2(k^2)$ and $\hat{\Pi}_L(k^2)$ may be subtracted at a renormalization point, say, $k^2 = \mu^2$. Thus, we can write [13-17]

$$\begin{gathered}\hat{\Omega}(k^2) = \hat{\Omega}(\mu^2) + \hat{\Omega}^c(k^2), \;\; \Pi_1(k^2) = \Pi_1(\mu^2) + \Pi_1^c(k^2), \\ \Pi_2(k^2) = \Pi_2(\mu^2) + \Pi_2^c(k^2), \;\; \hat{\Pi}_L(k^2) = \hat{\Pi}_L(\mu^2) + \hat{\Pi}_L^c(k^2)\end{gathered} \tag{4.24}$$

where $\hat{\Omega}(\mu^2)$, $\Pi_1(\mu^2)$, $\Pi_2(\mu^2)$, $\hat{\Pi}_L(\mu^2)$ and $\Omega^c(k^2)$, $\Pi_1^c(k^2)$, $\Pi_2^c(k^2)$, $\hat{\Pi}_L^c(k^2)$ are respectively the divergent parts and the finite parts of the functions $\Omega(k^2)$, $\Pi_1(k^2)$, $\Pi_2(k^2)$ and $\hat{\Pi}_L(k^2)$. The divergent parts can be absorbed in the following renormalization constants defined by [13-17]

$$\begin{gathered}\tilde{Z}_3^{-1} = 1 + \hat{\Omega}(\mu^2), \;\; Z_3^{-1} = 1 + \Pi_1(\mu^2), \;\; Z_3'^{-1} = 1 + \hat{\Pi}_L(\mu^2), \\ Z_M^{-1} = \sqrt{Z_3[1-\Pi_2(\mu^2)]} = \sqrt{[1-\Pi_1(\mu^2)][1-\Pi_2(\mu^2)]}\end{gathered} \tag{4.25}$$

where $Z_3$ and $\tilde{Z}_3$ are the renormalization constants of gluon and ghost particle propagators respectively, $Z_3'$ is the additional renormalization constant of the longitudinal part of gluon propagator and $Z_M$ is the renormalization constant of gluon mass. With the above definitions of the renormalization constants, on inserting Eq. (4.24) into Eqs. (4.11) and (4.22) , the ghost particle propagator and gluon propagator can be renormalized, respectively, in such a manner

$$i\Delta^{ab}(k) = \tilde{Z}_3 i\Delta_R^{ab}(k) \tag{4.26}$$

and

$$iD_{\mu\nu}^{ab}(k) = Z_3 iD_{R\mu\nu}^{\;ab}(k) \tag{4.27}$$

where

$$i\Delta_R^{ab}(k) = \frac{-i\delta^{ab}}{k^2[1+\Omega_R(k^2)] - \sigma_R^2 + i\varepsilon} \tag{4.28}$$

and

$$iD_{R\mu\nu}^{ab}(k) = -i\delta^{ab}\{\frac{g_{\mu\nu} - k_\mu k_\nu/k^2}{k^2 - M_R^2 + \Pi_R^T(k^2) + i\varepsilon} + \frac{Z_3'\alpha_R k_\mu k_\nu/k^2}{k^2[1+\Pi_R^L(k^2)] - \overline{\sigma}_R^2 + i\varepsilon}\} \tag{4.29}$$

are the renormalized propagators in which $M_R$, $\overline{\sigma}_R$ and $\widetilde{\sigma}_R$ are the renormalized masses, $\alpha_R$ is the renormalized gauge parameter, $\Omega_R(k^2), \Pi_R^T(k^2)$ and $\Pi_R^L(k^2)$ denote the finite corrections coming from the loop diagrams. They are defined as

$$\begin{gathered}M_R = Z_M^{-1}M, \;\; \alpha_R = Z_3^{-1}\alpha, \;\; \overline{\sigma}_R = \sqrt{Z_3'}\sigma, \;\; \sigma_R = \sqrt{\widetilde{Z}_3}\sigma, \\ \Omega_R(k^2) = \tilde{Z}_3\hat{\Omega}^c(k^2), \;\; \Pi_R^T(k^2) = Z_3[k^2\Pi_1^c(k^2) + M^2\Pi_2^c(k^2)], \;\; \Pi_R^L(k^2) = Z_3'\hat{\Pi}_L^c(k^2).\end{gathered} \tag{4.30}$$

The finite corrections above are zero at the renormalization point $\mu$. As we see from Eq. (4.29), the longitudinal part of the gluon propagator, except for in the Landau gauge, needs to be renormalized and has an extra renormalization constant $Z_3'$. This fact coincides with the general property of the massive vector boson propagator (see Ref. (16), Chap.V). From Eqs. (4.23)-(4.25) , it is easy to find that the longitudinal part in Eq. ( 4.22) can be renormalized as

$$\frac{\alpha}{k^2[1+\hat{\Pi}_L(k^2)] - \sigma^2 + i\varepsilon} = Z_3\alpha_R[1+\Omega_R(k^2)]\Delta_R(k^2) \tag{4.31}$$

where

$$\Delta_R(k^2) = \frac{1}{k^2[1+\Omega_R(k^2)] - \sigma_R^2 + i\varepsilon} \tag{4.32}$$

which appears in Eq. (4.28) and the renormalization constant $Z_3'$ can be expressed as

$$Z_3' = [1 + \frac{\sigma_R^2}{\mu^2}\frac{(1-\tilde{Z}_3)}{\tilde{Z}_3}]^{-1}. \tag{4.33}$$

If choosing $\mu = \sigma_R$, we have

$$Z_3' = \tilde{Z}_3. \tag{4.34}$$

## V. GLUON THREE-LINE VERTEX

The aim of this section is to derive the W-T identity satisfied by the gluon three-line proper vertex and discuss its renormalization. For this purpose, we first derive a W-T identity satisfied by the gluon three-point Green function. Let us begin with the derivation from the W-T identity in Eq. (4.1) and the ghost equation in Eq. (4.2). By taking successive differentiations of Eq. (4.1) with respect to the sources $J_\nu^b(y)$ and $J_\lambda^c(z)$ and then setting the sources to vanish, one may obtain the W-T identity obeyed by the gluon three-point Green function which is written in the operator form as follows

$$\begin{aligned}\frac{1}{\alpha}\partial_x^\mu G_{\mu\nu\lambda}^{abc}(x,y,z) = &< 0^+|T^*[\hat{\bar{C}}^a(x)\hat{D}_\nu^{bd}(y)\hat{C}^d(y)\hat{A}_\lambda^c(z)]|0^- > \\ &+ < 0^+|T^*[\hat{\bar{C}}^a(x)\hat{A}_\nu^b(y)\hat{D}_\lambda^{cd}(z)\hat{C}^d(z)]|0^- >\end{aligned} \tag{5.1}$$

where

$$G_{\mu\nu\lambda}^{abc}(x,y,z) =< 0^+|T[\hat{A}_\mu^a(x)\hat{A}_\nu^b(y)\hat{A}_\lambda^c(z)]|0^- > \tag{5.2}$$

is the three-point Green function mentioned above. The identity in Eq. (5.1) will be simplified by a ghost equation which may be derived by differentiating Eq. (4.2) with respect to the source $J_\lambda^c(z)$

$$\begin{aligned}&\partial_x^\mu < 0^+|T^*\{\hat{D}_\mu^{ad}(x)\hat{C}^d(x)\hat{\bar{C}}^b(y)\hat{A}_\lambda^c(z)\}|0^- > \\ &+\sigma^2 < 0^+|T[\hat{C}^a(x)\hat{\bar{C}}^b(y)\hat{A}_\lambda^c(z)]|0^-] >= 0.\end{aligned} \tag{5.3}$$

Taking derivatives of Eq. (5.1) with respect to $y$ and $z$ and employing Eq. (5.3), we get

$$\partial_x^\mu\partial_y^\nu\partial_z^\lambda G_{\mu\nu\lambda}^{abc}(x,y,z) = \alpha\sigma^2\{\partial_y^\nu G^{cab}{}_\nu(z,x,y) + \partial_z^\lambda G^{bac}{}_\lambda(y,x,z)\} \tag{5.4}$$

where

$$G^{abc}{}_\mu(x,y,z) =< 0^+|T\{\hat{C}^a(x)\hat{\bar{C}}^b(y)\hat{A}_\mu^c(z)\}|0^- > . \tag{5.5}$$

In the Landau gauge or the zero-mass limit ($\sigma = 0$), Eq. (5.4) reduces to

$$\partial_x^\mu\partial_y^\nu\partial_z^\lambda G_{\mu\nu\lambda}^{abc}(x,y,z) = 0 \tag{5.6}$$

which shows the transversity of the Green function. From Eq. (5.4), we may derive a W-T identity for the gluon three-line vertex. For this purpose, it is necessary to use the following one-particle-irreducible decompositions of the Green functions which can easily be obtained by the well-known procedure [13-16]

$$\begin{aligned}&G_{\mu\nu\lambda}^{abc}(x,y,z) = \int d^4x'd^4y'd^4z'iD_{\mu\mu'}^{aa'}(x-x') \\ &\times iD_{\nu\nu'}^{bb'}(y-y')iD_{\lambda\lambda'}^{cc'}(z-z')\Gamma_{a'b'c'}^{\mu'\nu'\lambda'}(x',y',z')\end{aligned} \tag{5.7}$$

and

$$\begin{aligned}G^{abc}{}_\nu(x,y,z) = &\int d^4x'd^4y'd^4z'i\Delta^{aa'}(x-x')\Gamma^{a'b'c',\nu'}(x',y',z') \\ &\times i\Delta^{b'b}(y'-y)iD_{\nu'\nu}^{c'c}(z'-z)\end{aligned} \tag{5.8}$$

where $iD_{\mu\mu'}^{aa'}(x-x')$ and $i\Delta^{aa'}(x-x')$ are respectively the gluon and the ghost particle propagators discussed in the preceding section, $\Gamma_{abc}^{\mu\nu\lambda}(x,y,z)$ and $\Gamma^{abc}_\lambda(x,y,z)$ are the three-line gluon proper vertex and the three-line ghost-gluon proper vertex respectively. They are defined as [13-16]

$$\Gamma^{\mu\nu\lambda}_{abc}(x,y,z) = i\frac{\delta^3\Gamma}{\delta A^a_\mu(x)\delta A^b_\nu(y)\delta A^c_\lambda(z)}|_{J=0} \tag{5.9}$$

and

$$\Gamma^{abc}{}_{\lambda}(x,y,z) = \frac{\delta^3\Gamma}{i\delta\bar{C}^a(x)\delta C^b(y)\delta A^{c\lambda}(z)}|_{J=0} \tag{5.10}$$

where $J$ stands for all the external sources. Substituting Eqs. (5.7) and (5.8) into Eq. (5.4) and transforming Eq. (5.4) into the momentum space, one can derive an identity which establishes the relation between the longitudinal part of three-line gluon vertex and the three-line ghost-gluon vertex as follows

$$\begin{aligned} p^\mu q^\nu k^\lambda \Lambda^{abc}_{\mu\nu\lambda}(p,q,k) = -\tfrac{\sigma^2}{\alpha}\chi(p^2)[\chi(k^2)q^\nu\Lambda^{cab}{}_{\nu}(k,p,q) \\ +\chi(q^2)k^\lambda\Lambda^{bac}{}_{\lambda}(q,p,k)] \end{aligned} \tag{5.11}$$

where we have defined

$$\begin{aligned} \Gamma^{abc}_{\mu\nu\lambda}(p,q,k) = (2\pi)^4\delta^4(p+q+k)\Lambda^{abc}_{\mu\nu\lambda}(p,q,k), \\ \Gamma^{abc}{}_{\lambda}(p,q,k) = (2\pi)^4\delta^4(p+q+k)\Lambda^{abc}{}_{\lambda}(p,q,k) \end{aligned} \tag{5.12}$$

and

$$\begin{aligned} \chi(p^2) = \{k^2[1+\hat{\Pi}_L(p^2)] - \sigma^2 + i\varepsilon\}\{k^2[1+\hat{\Omega}(p^2)] - \sigma^2 + i\varepsilon\}^{-1} \\ = [1+\widehat{\Omega}(k^2)]^{-1} \end{aligned} \tag{5.13}$$

here $\hat{\Pi}_L(p^2)$ and $\hat{\Omega}(p^2)$ are the self-energies defined in Eqs. (4.12) and (4.21). The second equality is obtained by inserting the relation in Eq. (4.23) into the first equality.

Obviously, in the Landau gauge, Eq. (5.11) reduces to

$$p^\mu q^\nu k^\lambda \Lambda^{abc}_{\mu\nu\lambda}(p,q,k) = 0 \tag{5.14}$$

which implies that the vertex is transverse in this case. In the lowest order approximation, owing to

$$\chi(p^2) = 1 \tag{5.15}$$

and

$$\Lambda^{(0)abc}{}_{\mu}(p,q,k) = gf^{abc}p_\mu \tag{5.16}$$

where $f^{abc}$ are the structure constants of the gauge group, the right hand side (RHS) of Eq. (5.11) vanishes, therefore, we have

$$p^\mu q^\nu k^\lambda \Lambda^{(0)abc}{}_{\mu\nu\lambda}(p,q,k) = 0. \tag{5.17}$$

This result is consistent with that for the bare three-line gluon vertex given by the Feynman rule.

Now, let us discuss renormalization of the three-line gluon vertex. From the renormalization of the gluon and ghost particle propagators described in Eqs. (4.26) and (4.27) and the definitions of the propagators written in Eqs. (4.6) and (4.7), one can see

$$\begin{aligned} A^{a\mu}(x) = \sqrt{Z_3}A^{a\mu}_R(x), \\ C^a(x) = \sqrt{\tilde{Z}_3}C^a_R(x),\ \bar{C}^a(x) = \sqrt{\tilde{Z}_3}\bar{C}^a_R(x) \end{aligned} \tag{5.18}$$

(hereafter the subscript $R$ marks renormalized quantities). According to above relations and the definitions given in Eqs. (5.9), (5.10) and (5.12), we find

$$\begin{aligned} \Lambda^{abc}_{\mu\nu\lambda}(p,q,k) = Z_3^{-3/2}\Lambda_{R\mu\nu\lambda}{}^{abc}(p,q,k), \\ \Lambda^{abc}{}_{\lambda}(p,q,k) = \tilde{Z}_3^{-1}Z_3^{-1/2}\Lambda_R{}^{abc}{}_{\lambda}(p,q,k). \end{aligned} \tag{5.19}$$

Applying these relations, the renormalized version of the identity written in Eq. (5.11) will be

$$p^\mu q^\nu k^\lambda \Lambda_{R\mu\nu\lambda}^{abc}(p,q,k) = -\frac{\sigma_R^2}{\alpha_R}\chi_R(p^2)[\chi_R(k^2) q^\nu \Lambda_R{}^{cab}_{\ \ \nu}(k,p,q) + \chi_R(q^2) k^\lambda \Lambda_R{}^{bac}_{\ \ \lambda}(q,p,k)] \tag{5.20}$$

where $\alpha_R$ and $\widetilde{\sigma}_R$ were defined in Eq. (4.30) and

$$\chi_R(k^2) = \frac{1}{1+\Omega_R(k^2)]} \tag{5.21}$$

is the renormalized expression of the function $\chi(k^2)$. In the above, we have considered

$$\chi(k^2) = \widetilde{Z}_3 \chi_R(k^2) \tag{5.22}$$

which follows from $\hat{\Omega}(k^2) = \hat{\Omega}(\mu^2) + \hat{\Omega}^c(k^2)$, $\tilde{Z}_3^{-1} = 1 + \hat{\Omega}(\mu^2)$ and $\Omega_R(k^2) = \tilde{Z}_3 \hat{\Omega}^c(k^2)$ defined in the preceding section. At the renormalization point chosen to be $p^2 = q^2 = k^2 = \mu^2$, we see, $\chi_R(\mu^2) = 1$. In this case, the renormalized ghost-gluon vertex takes the form of the bare vertex so that the RHS of Eq. (5.20) vanishes, therefore, we have

$$p^\mu q^\nu k^\lambda \Lambda_{R\mu\nu\lambda}^{abc}(p,q,k)|_{p^2=q^2=k^2=\mu^2} = 0. \tag{5.23}$$

Ordinarily, one is interested in discussing the renormalization of such three-line vertices that they are defined from the vertices defined in Eqs. (5.9) and (5.10) by extracting a coupling constant $g$. These vertices are denoted by $\tilde{\Lambda}_{\mu\nu\lambda}^{abc}(p,q,k)$ and $\tilde{\Lambda}^{abc}_{\ \ \lambda}(p,q,k)$. Commonly, they are renormalized in such a fashion [13-17]

$$\begin{aligned} \tilde{\Lambda}_{\mu\nu\lambda}^{abc}(p,q,k) &= Z_1^{-1} \widetilde{\Lambda}_{R\mu\nu\lambda}^{abc}(p,q,k), \\ \tilde{\Lambda}^{abc}_{\ \ \lambda}(p,q,k) &= \widetilde{Z}_1^{-1} \widetilde{\Lambda}_R^{abc}{}_{\lambda}(p,q,k) \end{aligned} \tag{5.24}$$

where $Z_1$ and $\widetilde{Z}_1$ are referred to as the renormalization constants for the gluon three-line vertex and the ghost-gluon vertex, respectively. It is clear that the W-T identity shown in Eq. (5.11) also holds for the vertices $\tilde{\Lambda}_{\mu\nu\lambda}^{abc}(p,q,k)$ and $\tilde{\Lambda}^{abc}_{\ \ \lambda}(p,q,k)$. So, when the vertices $\Lambda_{\mu\nu\lambda}^{abc}(p,q,k)$ and $\Lambda^{abc}_{\ \ \lambda}(p,q,k)$ in Eqs. (5.11) are replaced by $\tilde{\Lambda}_{\mu\nu\lambda}^{abc}(p,q,k)$ and $\tilde{\Lambda}^{abc}_{\ \ \lambda}(p,q,k)$ respectively and then Eq. (5.24) is inserted to such an identity, we obtain a renormalized version of the identity as follows

$$\begin{aligned} p^\mu q^\nu k^\lambda \tilde{\Lambda}_{R\mu\nu\lambda}^{abc}(p,q,k) = &-\frac{Z_1 \tilde{Z}_3}{Z_3 \tilde{Z}_1}\frac{\widetilde{\sigma}_R^2}{\alpha_R}\chi_R(p^2)[\chi_R(k^2) \\ &\times q^\nu \tilde{\Lambda}_R{}^{cab}_{\ \ \nu}(k,p,q) + \chi_R(q^2) k^\lambda \tilde{\Lambda}_R{}^{bac}_{\ \ \lambda}(q,p,k)]. \end{aligned} \tag{5.25}$$

When multiplying the both sides of Eq. (5.25) with a renormalized coupling constant $g_R$ and absorbing it in the vertices, noticing

$$\begin{aligned} \Lambda_{R\mu\nu\lambda}^{abc}(p,q,k) &= g_R \widetilde{\Lambda}_{R\mu\nu\lambda}^{abc}(p,q,k), \\ \Lambda_R^{abc}{}_{\lambda}(p,q,k) &= g_R \widetilde{\Lambda}_R^{abc}{}_{\lambda}(p,q,k), \end{aligned} \tag{5.26}$$

we have

$$\begin{aligned} p^\mu q^\nu k^\lambda \Lambda_{R\mu\nu\lambda}^{abc}(p,q,k) = &-\frac{Z_1 \tilde{Z}_3}{Z_3 \tilde{Z}_1}\frac{\sigma_R^2}{\alpha_R}\chi_R(p^2)[\chi_R(k^2) \\ &\times q^\nu \Lambda_R{}^{cab}_{\ \ \nu}(k,p,q) + \chi_R(q^2) k^\lambda \Lambda_R{}^{bac}_{\ \ \lambda}(q,p,k)]. \end{aligned} \tag{5.27}$$

In comparison of Eq. (5.27) with Eq. (5.20), we see, except for the factor $Z_1 \tilde{Z}_3 Z_3^{-1} \tilde{Z}_1^{-1}$, the both identities are identical to each other. From this observation, we deduce

$$\frac{Z_1}{Z_3} = \frac{\tilde{Z}_1}{\tilde{Z}_3}. \tag{5.28}$$

This is the S-T identity which coincides with the one given in the massless QCD [19, 20].

## VI. GLUON FOUR-LINE VERTEX

By the similar procedure as deriving Eqs. (5.1) and (5.3), the W-T identity obeyed by the gluon four-point Green function may be derived by differentiating Eq. (4.1) with respect to the sources $J_\mu^b(y), J_\lambda^c(z)$ and $J_\tau^d(u)$. The result represented in the operator form is as follows

$$\begin{array}{c}\frac{1}{\alpha}\partial_x^\mu G_{\mu\nu\lambda\tau}^{abcd}(x,y,z,u)\\ =< 0^+|T^*[\hat{\bar{C}}^a(x)\hat{D}_\nu^{be}(y)\hat{C}^e(y)\hat{A}_\lambda^c(z)\hat{A}_\tau^d(u)]|0^- >\\ + < 0^+|T^*[\hat{\bar{C}}^a(x)\hat{A}_\nu^b(y)\hat{D}_\lambda^{ce}(z)\hat{C}^e(z)\hat{A}_\tau^d(u)]|0^- >\\ + < 0^+|T^*[\hat{\bar{C}}^a(x)\hat{A}_\nu^b(y)\hat{A}_\lambda^c(z)\hat{D}_\tau^{de}(u)\hat{C}^e(u)]|0^- >\end{array} \tag{6.1}$$

where

$$G_{\mu\nu\lambda\tau}^{abcd}(x,y,z,u) =< 0^+|T[\hat{A}_\mu^a(x)\hat{A}_\nu^b(y)\hat{A}_\lambda^c(z)\hat{A}_\tau^d(u)]|0^- > \tag{6.2}$$

is the gluon four-point Green function. The accompanying ghost equation may be obtained by differentiating Eq. (4.2) with respect to the sources $J_\lambda^c(z)$ and $J_\tau^d(u)$. The result is

$$\begin{array}{c}\partial_x^\mu < 0^+|T^*[\hat{D}_\mu^{ae}(x)\hat{C}^e(x)\hat{\bar{C}}^b(y)\hat{A}_\lambda^c(z)\hat{A}_\tau^d(u)]|0^- >\\ +\sigma^2 G_{\lambda\tau}^{abcd}(x,y,z,u) = -\delta^{ab}\delta^4(x-y)D_{\lambda\tau}^{cd}(z-u)\end{array} \tag{6.3}$$

where

$$G_{\lambda\tau}^{abcd}(x,y,z,u) =< 0^+|T[\hat{C}^a(x)\hat{\bar{C}}^b(y)\hat{A}_\lambda^c(z)\hat{A}_\tau^d(u)]|0^- > \tag{6.4}$$

is the four-point gluon-ghost particle Green function. Differentiation of Eq. (6.1) with respect to the coordinates $y$, $z$ and $u$ and use of Eq. (6.3) lead to

$$\begin{array}{c}\frac{1}{\alpha}\partial_x^\mu\partial_y^\nu\partial_z^\lambda\partial_u^\tau G_{\mu\nu\lambda\tau}^{abcd}(x,y,z,u) = \delta^{ab}\delta^4(x-y)\partial_z^\lambda\partial_u^\tau D_{\lambda\tau}^{cd}(z-u)\\ +\delta^{ac}\delta^4(x-z)\partial_y^\nu\partial_u^\tau D_{\nu\tau}^{bd}(y-u) + \delta^{ad}\delta^4(x-u)\partial_y^\nu\partial_z^\lambda D_{\nu\lambda}^{bc}(y-z)\\ +\sigma^2\{\partial_z^\lambda\partial_u^\tau G_{\lambda\tau}^{bacd}(y,x,z,u) + \partial_y^\nu\partial_u^\tau G_{\nu\tau}^{cabd}(z,x,y,u)\\ +\partial_y^\nu\partial_z^\lambda G_{\nu\lambda}^{dabc}(u,x,y,z)\}.\end{array} \tag{6.5}$$

It is noted that the four-point Green functions appearing in the above equations are unconnected. Their decompositions to connected Green functions are not difficult to be found by making use of the relation between the generating functionals $Z$ for the full Green functions and $W$ for the connected Green functions as written in Eq. (3.9). The result is

$$\begin{array}{c}G_{\mu\nu\lambda\tau}^{abcd}(x,y,z,u) = G_{\mu\nu\lambda\tau}^{abcd}(x,y,z,u)_c - D_{\mu\nu}^{ab}(x-y)D_{\lambda\tau}^{cd}(z-u)\\ -D_{\mu\lambda}^{ac}(x-z)D_{\nu\tau}^{bd}(y-u) - D_{\mu\tau}^{ad}(x-u)D_{\nu\lambda}^{bc}(y-z)\end{array} \tag{6.6}$$

and

$$G_{\lambda\tau}^{abcd}(x,y,z,u) = G_{\lambda\tau}^{abcd}(x,y,z,u)_c - \Delta^{ab}(x-y)D_{\lambda\tau}^{cd}(z-u). \tag{6.7}$$

The first terms marked by the subscript "$c$" in Eqs. ( 6.6) and (6.7) are connected Green functions. When inserting Eqs. (6.6) and (6.7) into Eq. (6.5) and using the W-T identity in Eq. (4.5), one may find

$$\begin{array}{c}\partial_x^\mu\partial_y^\nu\partial_z^\lambda\partial_u^\tau G_{\mu\nu\lambda\tau}^{abcd}(x,y,z,u)_c = \alpha\sigma^2\{\partial_y^\nu\partial_z^\lambda G_{\nu\lambda}^{dabc}(u,x,y,z)_c\\ +\partial_y^\nu\partial_u^\tau G_{\nu\tau}^{cabd}(z,x,y,u)_c + \partial_z^\lambda\partial_u^\tau G_{\lambda\tau}^{bacd}(y,x,z,u)_c\}.\end{array} \tag{6.8}$$

This is the W-T identity satisfied by the connected four-point Green functions. In the Landau gauge, we have

$$\partial_x^\mu\partial_y^\nu\partial_z^\lambda\partial_u^\tau G_{\mu\nu\lambda\tau}^{abcd}(x,y,z,u)_c = 0 \tag{6.9}$$

which shows the transversity of the Green function.

The W-T identity for the four-line proper gluon vertex may be derived from Eq. (6.8) with the help of the following one-particle-irreducible decompositions of the connected Green functions which can easily be found by the standard procedure [13-16].

$$
\begin{array}{c}
G^{abcd}_{\mu\nu\lambda\tau}(x_1,x_2,x_3,x_4)_c \\
=\int\prod\limits_{i=1}^{4}d^4y_iD^{aa'}_{\mu\mu'}(x_1-y_1)D^{bb'}_{\nu\nu'}(x_2-y_2)\Gamma^{\mu'\nu'\lambda'\tau'}_{a'b'c'd'}(y_1,y_2,y_3,y_4) \\
\times D^{c'c}_{\lambda'\lambda}(y_3-x_3)D^{d'd}_{\tau'\tau}(y_4-x_4) \\
+i\int\prod\limits_{i=1}^{4}d^4y_id^4z_i\{D^{aa'}_{\mu\mu'}(x_1-y_1)D^{bb'}_{\nu\nu'}(x_2-y_2)\Gamma^{\mu'\nu'\rho}_{a'b'e}(y_1,y_2,y_3) \\
\times D^{ee'}_{\rho\rho'}(y_3-z_1)\Gamma^{\rho'\lambda'\tau'}_{e'c'd'}(z_1,z_2,z_3)D^{c'c}_{\lambda'\lambda}(z_2-x_3)D^{d'd}_{\tau'\tau}(z_3-x_4) \\
+D^{aa'}_{\mu\mu'}(x_1-y_1)D^{cc'}_{\lambda\lambda'}(x_3-y_2)\Gamma^{\mu'\lambda'\rho}_{a'c'e}(y_1,y_2,y_3)D^{ee'}_{\rho\rho'}(y_3-z_1) \\
\times\Gamma^{\rho'\nu'\tau'}_{e'b'd'}(z_1,z_2,z_3)D^{b'b}_{\nu'\nu}(z_2-x_2)D^{d'd}_{\tau'\tau}(z_3-x_4) \\
+D^{bb'}_{\nu\nu'}(x_2-y_1)D^{cc'}_{\lambda\lambda'}(x_3-y_2)\Gamma^{\nu'\lambda'\rho}_{b'c'e}(y_1,y_2,y_3)D^{ee'}_{\rho\rho'}(y_3-z_1) \\
\times\Gamma^{\rho'\mu'\tau'}_{e'a'd'}(z_1,z_2,z_3)D^{a'a}_{\mu'\mu}(z_2-x_1)D^{d'd}_{\tau'\tau}(z_3-x_4)\}
\end{array}
\tag{6.10}
$$

and

$$
\begin{array}{c}
G^{abcd}_{\lambda\tau}(x_1,x_2,x_3,x_4)_c \\
=\int\prod\limits_{i=1}^{4}d^4y_i\Delta^{aa'}(x_1-y_1)\Gamma_{a'b'c'd'}{}^{\lambda'\tau'}(y_1,y_2,y_3,y_4)\Delta^{b'b}(y_2-x_2) \\
\times D^{c'c}_{\lambda'\lambda}(y_3-x_3)D^{d'd}_{\tau'\tau}(y_4-x_4) \\
+i\int\prod\limits_{i=1}^{4}d^4y_id^4z_i\{\Delta^{aa'}(x_1-y_1)\Gamma_{a'ed'}{}^{\tau'}(y_1,y_2,y_3)\Delta^{ee'}(y_2-z_1) \\
\times D^{d'd}_{\tau'\tau}(y_3-x_4)\Gamma_{e'b'c'}{}^{\lambda'}(z_1,z_2,z_3)\Delta^{b'b}(z_2-x_2)D^{c'c}_{\lambda'\lambda}(z_3-x_3) \\
+\Delta^{aa'}(x_1-y_1)\Gamma_{a'ec'}{}^{\lambda'}(y_1,y_2,y_3)\Delta^{ee'}(y_2-z_1)D^{c'c}_{\lambda'\lambda}(y_3-x_3) \\
\times\Gamma_{e'b'd'}{}^{\tau'}(z_1,z_2,z_3)\Delta^{b'b}(z_2-x_2)D^{d'd}_{\tau'\tau}(z_3-x_4) \\
+\Delta^{aa'}(x_1-y_1)\Gamma_{a'b'e}{}^{\rho}(y_1,y_2,y_3)\Delta^{b'b}(y_2-x_2)D^{ee'}_{\rho\rho'}(y_3-z_1) \\
\times\Gamma^{\rho'\lambda'\tau'}_{e'c'd'}(z_1,z_2,z_3)D^{c'c}_{\lambda'\lambda}(z_2-x_3)D^{d'd}_{\tau'\tau}(z_3-x_4)\}
\end{array}
\tag{6.11}
$$

where $\Gamma^{abcd}_{\mu\nu\lambda\tau}(x_1,x_2,x_3,x_4)$ is the four-line gluon proper vertex and $\Gamma^{abcd}_{\lambda\tau}(x_1,x_2,x_3,x_4)$ is the four-line ghost-gluon proper vertex. They are defined as [13-16]

$$
\begin{array}{c}
\Gamma^{abcd}_{\mu\nu\lambda\tau}(x_1,x_2,x_3,x_4)=i\frac{\delta^4\Gamma}{\delta A^{a\mu}(x_1)\delta A^{b\nu}(x_2)\delta A^{c\lambda}(x_3)\delta A^{d\tau}(x_4)}|_{J=0}, \\
\Gamma^{abcd}_{\lambda\tau}(x_1,x_2,x_3,x_4)=\frac{\delta^4\Gamma}{i\delta\bar{C}^a(x_1)\delta C^b(x_2)\delta A^{c\lambda}(x_3)\delta A^{d\tau}(x_4)}|_{J=0}.
\end{array}
\tag{6.12}
$$

When substituting Eqs. (6.10) and (6.11) into Eq. (6.8) and transforming Eq. (6.8) into the momentum space, one can find the following identity satisfied by the four-line proper gluon vertex

$$
\begin{array}{c}
k_1^\mu k_2^\nu k_3^\lambda k_4^\tau\Lambda^{abcd}_{\mu\nu\lambda\tau}(k_1,k_2,k_3,k_4)=\Psi\begin{pmatrix} a & b & c & d \\ k_1 & k_2 & k_3 & k_4\end{pmatrix} \\
+\Psi\begin{pmatrix} a & c & d & b \\ k_1 & k_3 & k_4 & k_2\end{pmatrix}+\Psi\begin{pmatrix} a & d & b & c \\ k_1 & k_4 & k_2 & k_3\end{pmatrix}
\end{array}
\tag{6.13}
$$

where

$$
\begin{array}{c}
\Psi\begin{pmatrix} a & b & c & d \\ k_1 & k_2 & k_3 & k_4\end{pmatrix} \\
=-ik_1^\mu k_2^\nu\Lambda^{abe}_{\mu\nu\sigma}(k_1,k_2,-(k_1+k_2))D^{\sigma\rho}_{ef}(k_1+k_2)k_3^\lambda k_4^\tau\Lambda^{fcd}_{\rho\lambda\tau}(-(k_3+k_4),k_3,k_4) \\
+\frac{i\sigma^2}{\alpha}\chi(k_1^2)\chi(k_2^2)[ik_3^\lambda k_4^\tau\Lambda^{bacd}_{\lambda\tau}(k_2,k_1,k_3,k_4) \\
-\Lambda^{bae}_{\sigma}(k_2,k_1,-(k_1+k_2))D^{\sigma\rho}_{ef}(k_1+k_2)k_3^\lambda k_4^\tau\Lambda^{fcd}_{\rho\lambda\tau}(-(k_3+k_4),k_3,k_4) \\
-k_4^\tau\Lambda^{bed}{}_{\tau}(k_2,-(k_2+k_4),k_4)\Delta^{ef}(k_2+k_4)k_3^\lambda\Lambda^{fac}{}_{\lambda}(-(k_1+k_3),k_1,k_3) \\
-k_3^\lambda\Lambda^{bec}{}_{\lambda}(k_2,-(k_2+k_3),k_3)\Delta^{ef}(k_2+k_3)k_4^\tau\Lambda^{fad}{}_{\tau}(-(k_1+k_4),k_1,k_4)].
\end{array}
\tag{6.14}
$$

The second and third terms in Eq .(6.13) can be written out from Eq. (6.14) through cyclic permutations. In the above, we have defined

$$
\begin{array}{c}
\Gamma^{abcd}_{\mu\nu\lambda\tau}(k_1,k_2,k_3,k_4)=(2\pi)^4\delta^4(\sum_{i=1}^4k_i)\Lambda^{abcd}_{\mu\nu\lambda\tau}(k_1,k_2,k_3,k_4), \\
\Gamma^{abcd}_{\lambda\tau}(k_1,k_2,k_3,k_4)=(2\pi)^4\delta^4(\sum_{i=1}^4k_i)\Lambda^{abcd}_{\lambda\tau}(k_1,k_2,k_3,k_4).
\end{array}
\tag{6.15}
$$

In the lowest order approximation, we have checked that except for the first term in Eq. (6.14) which was encountered in the massless theory, the remaining mass-dependent terms are cancelled out with the corresponding terms contained in the second and third terms in Eq. (6.13). Therefore, the identity in Eq. (6.13) leads to a result in the lowest order approximation which is consistent with the Feynman rule.

The renormalization of the four-line vertices is similar to that for the three-line vertices. From the definitions given in Eqs. (6.12), (6.15) and (5.18), it is clearly seen that the four-line vertices should be renormalized in such a manner

$$\begin{array}{c}\Lambda^{abcd}_{\mu\nu\lambda\tau}(k_1,k_2,k_3,k_4)=Z_3^{-2}\Lambda_{R\mu\nu\lambda\tau}^{abcd}(k_1,k_2,k_3,k_4),\\ \Lambda^{abcd}_{\lambda\tau}(k_1,k_2,k_3,k_4)=\tilde{Z}_3^{-1}Z_3^{-1}\Lambda_{R}{}^{abcd}_{\lambda\tau}(k_1,k_2,k_3,k_4).\end{array} \tag{6.16}$$

On inserting these relations into Eqs. (6.13) and (6.14), one can obtain a renormalized identity similar to Eq. (5.20), that is

$$\begin{array}{c}k_1^\mu k_2^\nu k_3^\lambda k_4^\tau\Lambda^{abcd}_{R\mu\nu\lambda\tau}(k_1,k_2,k_3,k_4)=\Psi_R\begin{pmatrix}a & b & c & d\\ k_1 & k_2 & k_3 & k_4\end{pmatrix}\\ +\Psi_R\begin{pmatrix}a & c & d & b\\ k_1 & k_3 & k_4 & k_2\end{pmatrix}+\Psi_R\begin{pmatrix}a & d & b & c\\ k_1 & k_4 & k_2 & k_3\end{pmatrix}\end{array} \tag{6.17}$$

where

$$\begin{array}{c}\Psi_R\begin{pmatrix}a & b & c & d\\ k_1 & k_2 & k_3 & k_4\end{pmatrix}\\ =-ik_1^\mu k_2^\nu\Lambda^{abe}_{R\mu\nu\sigma}(k_1,k_2,-(k_1+k_2))D^{\sigma\rho}_{Ref}(k_1+k_2)k_3^\lambda k_4^\tau\Lambda^{fcd}_{R\rho\lambda\tau}(-(k_3+k_4),k_3,k_4)\\ +\frac{i\sigma_R^2}{\alpha_R}\chi_R(k_1^2)\chi_R(k_2^2)[ik_3^\lambda k_4^\tau\Lambda_R^{bacd}{}_{\lambda\tau}(k_2,k_1,k_3,k_4)\\ -\Lambda_R^{bae}{}_\sigma(k_2,k_1,-(k_1+k_2))D^{\sigma\rho}_{Ref}(k_1+k_2)k_3^\lambda k_4^\tau\Lambda^{fcd}_{R\rho\lambda\tau}(-(k_3+k_4),k_3,k_4)\\ -k_4^\tau\Lambda_R^{bed}{}_\tau(k_2,-(k_2+k_4),k_4)\Delta_R^{ef}(k_2+k_4)k_3^\lambda\Lambda_R^{fac}{}_\lambda(-(k_1+k_3),k_1,k_3)\\ -k_3^\lambda\Lambda_R^{bec}{}_\lambda(k_2,-(k_2+k_3),k_3)\Delta_R^{ef}(k_2+k_3)k_4^\tau\Lambda_R^{fad}{}_\tau(-(k_1+k_4),k_1,k_4)].\end{array} \tag{6.18}$$

We can also define vertices $\tilde{\Lambda}^{abcd}_{\mu\nu\lambda\tau}(k_1,k_2,k_3,k_4)$ and $\tilde{\Lambda}^{abcd}_{\lambda\tau}(k_1,k_2,k_3,k_4)$ from the vertices $\Lambda^{abcd}_{\mu\nu\lambda\tau}(k_1,k_2,k_3,k_4)$ and $\Lambda^{abcd}_{\lambda\tau}(k_1,k_2,k_3,k_4)$ by taking out the coupling constant squared, respectively. The renormalization of these vertices are usually defined by [13-17]

$$\begin{array}{c}\tilde{\Lambda}^{abcd}_{\mu\nu\lambda\tau}(k_1,k_2,k_3,k_4)=Z_4^{-1}\tilde{\Lambda}_{R\mu\nu\lambda\tau}^{abcd}(k_1,k_2,k_3,k_4),\\ \tilde{\Lambda}^{abcd}_{\lambda\tau}(k_1,k_2,k_3,k_4)=\tilde{Z}_4^{-1}\tilde{\Lambda}_{R}{}^{abcd}_{\lambda\tau}(k_1,k_2,k_3,k_4).\end{array} \tag{6.19}$$

where $Z_4$ and $\tilde{Z}_4$ are the renormalization constants of the four-line gluon and ghost-gluon vertices respectively. Obviously, the identity in Eqs. (6.13) and (6.14) remains formally unchanged if we replace all the vertices $\Lambda_i$ in the identity with the ones $\widetilde{\Lambda}_i$ . Substituting Eqs. (5.24), (6.19), (4.26) and (4.27) into such an identity, one may write a renormalized identity similar to Eq. (5.25), that is

$$\begin{array}{c}k_1^\mu k_2^\nu k_3^\lambda k_4^\tau\widetilde{\Lambda}^{abcd}_{R\mu\nu\lambda\tau}(k_1,k_2,k_3,k_4)=\widetilde{\Psi}_R\begin{pmatrix}a & b & c & d\\ k_1 & k_2 & k_3 & k_4\end{pmatrix}\\ +\widetilde{\Psi}_R\begin{pmatrix}a & c & d & b\\ k_1 & k_3 & k_4 & k_2\end{pmatrix}+\widetilde{\Psi}_R\begin{pmatrix}a & d & b & c\\ k_1 & k_4 & k_2 & k_3\end{pmatrix}\end{array} \tag{6.20}$$

where

$$\begin{array}{c}\widetilde{\Psi}_R\begin{pmatrix}a & b & c & d\\ k_1 & k_2 & k_3 & k_4\end{pmatrix}\\ =\frac{Z_4Z_3}{Z_1^2}\{-ik_1^\mu k_2^\nu\widetilde{\Lambda}^{abe}_{R\mu\nu\sigma}(k_1,k_2,-(k_1+k_2))D^{\sigma\rho}_{Ref}(k_1+k_2)k_3^\lambda k_4^\tau\widetilde{\Lambda}^{fcd}_{R\rho\lambda\tau}(-(k_3+k_4),k_3,k_4)\}\\ +\frac{i\sigma_R^2}{\alpha_R}\chi_R(k_1^2)\chi_R(k_2^2)\{\frac{\widetilde{Z}_3Z_4}{Z_3\widetilde{Z}_4}ik_3^\lambda k_4^\tau\widetilde{\Lambda}_R^{bacd}{}_{\lambda\tau}(k_2,k_1,k_3,k_4)\\ -\frac{Z_4\widetilde{Z}_3}{Z_1\widetilde{Z}_1}\widetilde{\Lambda}_R^{bae}{}_\sigma(k_2,k_1,-(k_1+k_2))D^{\sigma\rho}_{Ref}(k_1+k_2)k_3^\lambda k_4^\tau\widetilde{\Lambda}^{fcd}_{R\rho\lambda\tau}(-(k_3+k_4),k_3,k_4)\\ -\frac{Z_4\widetilde{Z}_3^2}{Z_3\widetilde{Z}_1^2}[k_4^\tau\widetilde{\Lambda}_R^{bed}{}_\tau(k_2,-(k_2+k_4),k_4)\Delta_R^{ef}(k_2+k_4)k_3^\lambda\widetilde{\Lambda}_R^{fac}{}_\lambda(-(k_1+k_3),k_1,k_3)\\ +k_3^\lambda\widetilde{\Lambda}_R^{bec}{}_\lambda(k_2,-(k_2+k_3),k_3)\Delta_R^{ef}(k_2+k_3)k_4^\tau\widetilde{\Lambda}_R^{fad}{}_\tau(-(k_1+k_4),k_1,k_4)]\}.\end{array} \tag{6.21}$$

Multiplying the both sides of Eqs. (6.20) and (6.21) by $g_R^2$, according to the relations given in Eqs. (5.26) and in the following

$$\begin{aligned}\Lambda^{abcd}_{R\mu\nu\lambda\tau}(k_1,k_2,k_3,k_4) = g_R^2\tilde{\Lambda}^{abcd}_{R\mu\nu\lambda\tau}(k_1,k_2,k_3,k_4),\\ \Lambda^{abcd}_{R\ \ \lambda\tau}(k_1,k_2,k_3,k_4) = g_R^2\widetilde{\Lambda}_R{}^{abcd}_{\ \ \lambda\tau}(k_1,k_2,k_3,k_4),\end{aligned} \tag{6.22}$$

we have an identity which is of the same form as the identity in Eqs. (6.20) and (6.21) except that the vertices $\widetilde{\Lambda}^i_R$ in Eqs. (6.20) and (6.21) are all replaced by the vertices $\Lambda^i_R$. Comparing this identity with that written in Eqs. (6.17) and (6.18), one may find

$$\frac{Z_3Z_4}{Z_1^2}=1,\frac{\widetilde{Z}_3Z_4}{Z_3\widetilde{Z}_4}=1,\frac{Z_4\widetilde{Z}_3}{Z_1\widetilde{Z}_1}=1,\frac{Z_4\widetilde{Z}_3^2}{Z_3\widetilde{Z}_1^2}=1 \tag{6.23}$$

which lead to

$$\frac{Z_1}{Z_3}=\frac{\widetilde{Z}_1}{\widetilde{Z}_3}=\frac{Z_4}{Z_1},\frac{Z_1}{\widetilde{Z}_1}=\frac{Z_3}{\widetilde{Z}_3}=\frac{Z_4}{\widetilde{Z}_4} \tag{6.24}$$

This just is the S-T identity which is consistent with that given in Refs. (19) and (20) for the massless QCD.

## VII. QUARK-GLUON VERTEX AND QUARK PROPAGATOR

This section is used to derive the W-T identity for quark-gluon vertex and discuss its renormalization. First we derive a W-T identity satisfied by the quark-antiquark-gluon three-point Green function. This identity can easily be derived by differentiating the W-T identity in Eq. (3.8) or (3.10) with respect to the sources $\xi^b(z)$, $\eta(y)$ and $\overline{\eta}(x)$ and then setting all the sources to be zero. The result written in the operator form is as follows

$$\partial_z^\mu G^a_\mu(x,y,z)=i\alpha g[G_1^{ba}(x,y,y,z)T^b-T^bG_2^{ba}(x,y,x,z)] \tag{7.1}$$

where

$$G^a_\mu(x,y,z)=\left\langle 0^+\mid\widehat{\psi}(x)\widehat{\overline{\psi}}(y)\widehat{A}(z)\mid 0^-\right\rangle \tag{7.2}$$

is the quark-gluon three-point Green function,

$$G_1^{ba}(x,y,z)=\left\langle 0^+\mid\widehat{\psi}(x)\widehat{\overline{\psi}}(y)\widehat{C}^b(y)\widehat{\overline{C}}^a(z)\mid 0^-\right\rangle \tag{7.3}$$

and

$$G_2^{ba}(x,y,z)=\left\langle 0^+\mid\widehat{\psi}(x)\widehat{\overline{\psi}}(y)\widehat{C}^b(x)\widehat{\overline{C}}^a(z)\mid 0^-\right\rangle \tag{7.4}$$

are the quark-ghost particle mixed Green functions. The Green functions in Eqs. (7.3) and (7.4) are connected because a quark field and a ghost field are of a common coordinate.

The W-T identity for quark-gluon vertex can be derived from Eq. (7.1) with the help of one-particle irreducible decompositions of the Green functions shown in Eqs. (7.2)-(7.4). The decompositions can easily be obtained by the standard procedure [13-16]. The results are given in the following.

$$G^a_\mu(x,y,z)=\int d^4x'd^4y'd^4z'iS_F(x-x')\Gamma^{b\nu}(x',y',z')iS_F(y'-y)iD^{ba}_{\nu\mu}(z'-z) \tag{7.5}$$

where $D^{ba}_{\nu\mu}(z'-z)$ is the gluon propagator defined in Eq. (4.6),

$$iS_F(x-x')=\left\langle 0^+\mid\widehat{\psi}(x)\widehat{\overline{\psi}}(x')\mid 0^-\right\rangle \tag{7.6}$$

is the quark propagator and

$$\Gamma^{b\nu}(x',y',z')=\frac{\delta^3\Gamma}{i\delta\overline{\psi}(x')\delta\psi(y')\delta A^b_\nu(z')}\mid_{J=0} \tag{7.7}$$

is the quark-gluon proper vertex.

$$G_1^{ba}(x,y,z)=\int d^4x'd^4z'S_F(x-x')\gamma^{bc}(x',y,z')\Delta^{ca}(z'-z) \tag{7.8}$$

where $\Delta^{ca}(z'-z)$ is the ghost particle propagator defined in Eq. (4.7) and

$$\gamma_1^{bc}(x',y,z')=\int d^4ud^4v\Delta^{bd}(y-u)\Gamma^{cd}(x',v,u,z')S_F(v-y) \tag{7.9}$$

in which

$$\Gamma^{cd}(x',v,u,z')=i\frac{\delta^4\Gamma}{\delta\overline{\psi}(x')\delta\psi(v)\delta\overline{C}^c(u)\delta C^d(z')}\mid_{J=0} \tag{7.10}$$

is the quark-ghost vertex. Similarly,

$$G_2^{ba}(x,y,z)=\int d^4y'd^4z'\gamma^{bc}(x,y',z')S_F(y'-y)\Delta^{ca}(z'-z) \tag{7.11}$$

where

$$\gamma_2^{bc}(x,y',z')=\int d^4ud^4vS_F(x-u)\Delta^{bd}(x-v)\Gamma^{dc}(u,y',v,z'). \tag{7.12}$$

On substituting Eqs. (7.5), (7.8) and (7.11) into Eq. (7.1) and then transform Eq. (7.1) into the momentum space, we have

$$\begin{array}{c} S_F(p)\Gamma^{b\nu}(p,q,k)S_F(q)k^\mu D_{\mu\nu}^{ab}(k) \\ =-i\alpha g[S_F(p)\gamma_1^b(p,q,k)-\gamma_2^b(p,q,k)S_F(q)]\Delta^{ab}(k)] \end{array} \tag{7.13}$$

where we have defined

$$\begin{array}{c} \gamma_1^a(p,q,k)=\gamma_1^{ab}(p,q,k)T^b, \\ \gamma_2^a(p,q,k)=T^b\gamma_2^{ba}(p,q,k). \end{array} \tag{7.14}$$

Considering that the vertex functions $\Gamma^{b\nu}(p,q,k)$ and $\gamma_i^a(p,q,k)$ $(i=1,2)$ contain a common delta-function representing the energy-momentum conservation, we may set

$$\begin{array}{c} \Gamma^{a\mu}(p,q,k)=(2\pi)^4\delta^4(p-q+k)\Lambda^{a\mu}(p,q,k), \\ \gamma_i^a(p,q,k)=(2\pi)^4\delta^4(p-q+k)\widetilde{\gamma}_i^a(p,q,k) \end{array} \tag{7.15}$$

where $\Lambda^{a\mu}(p,q,k)$ and $\widetilde{\gamma}_i^a(p,q,k)$ are the new vertex functions in which $k=q-p$. Noticing the above relations and the expressions of gluon and ghost particle propagators as given in Eqs. (4.11) and (4.22), the W-T identity in Eq. (7.13) can be rewritten via the functions $\Lambda^{a\mu}(p,q,k)$ and $\widetilde{\gamma}_i^a(p,q,k)$ in the form

$$k_\mu\Lambda^{a\mu}(p,q,k)=ig\chi(k^2)[S_F^{-1}(p)\widetilde{\gamma}_2^a(p,q,k)-\widetilde{\gamma}_1^a(p,q,k)S_F^{-1}(q)] \tag{7.16}$$

where $\chi(k^2)$ was defined in Eq. (5.13).

Let us turn to discuss the renormalized form of the above W-T identity. It is well-known that the quark propagator can be expressed in the form

$$S_F(p)=\frac{1}{\mathbf{p}-m-\Sigma(p)+i\varepsilon} \tag{7.17}$$

where $\mathbf{p}=\gamma^\mu p_\mu$ and $\Sigma(p)$ denotes the quark self-energy. The above expression can easily be derived from the Dyson equation [29]. Usually, the quark propagator is renormalized in such a fashion

$$S_F(p)=Z_2S_F^R(p) \tag{7.18}$$

which implies

$$\psi(x)=\sqrt{Z_2}\psi_R(x),\ \overline{\psi}(x)=\sqrt{Z_2}\overline{\psi}_R(x). \tag{7.19}$$

From the relations in Eqs. (7.19) and (5.18), it is clearly seen that the vertex defined in Eq. (7.7) is renormalized as

$$\Gamma^{a\mu}(x,y,z) = Z_2^{-1} Z_3^{-\frac{1}{2}} \Gamma_R^{a\mu}(x,y,z) \tag{7.20}$$

which leads to

$$\Lambda^{a\mu}(p,q,k) = Z_2^{-1} Z_3^{-\frac{1}{2}} \Lambda_R^{a\mu}(p,q,k) \tag{7.21}$$

The functions $\widetilde{\gamma}_i^a(p,q,k)$ are, in general, divergent in the perturbative calculation. These functions are assumed to be renormalized in such a manner

$$\widetilde{\gamma}_i^a(p,q,k) = Z_\gamma^{-1} \widetilde{\gamma}_{iR}^a(p,q,k). \tag{7.22}$$

where $Z_\gamma$ is the renormalization constant of the functions $\widetilde{\gamma}_i^a(p,q,k)$. Based on the relations in Eqs. (5.22), (7.18), (7.21), and (7.22), Eq. (7.16) can be represented in terms of the renormalized quantities

$$k_\mu \Lambda_R^{a\mu}(p,q,k) = i g_R \chi_R(k^2)[S_F^{R-1}(p)\widetilde{\gamma}_{2R}^a(p,q,k) - \widetilde{\gamma}_{1R}^a(p,q,k) S_F^{R-1}(q)] \tag{7.23}$$

where $g_R$ is the renormalized coupling constant defined by

$$g_R = \widetilde{Z}_3 Z_3^{\frac{1}{2}} \widetilde{Z}_\gamma^{-1} g \tag{7.24}$$

It is well-known that

$$g_R = \widetilde{Z}_3 Z_3^{\frac{1}{2}} \widetilde{Z}_1^{-1} g \tag{7.25}$$

where $\widetilde{Z}_1$ is the ghost vertex renormalization constant as defined in Eq. (5.24). The relation in Eq. (7.25) ordinarily is determined from the renormalization of S-matrix elements. In comparison of Eq. (7.24) with Eq. (7.25), we see

$$\widetilde{Z}_\gamma = \widetilde{Z}_1 \tag{7.26}$$

which means that the functions $\widetilde{\gamma}_i^a(p,q,k)$ are renormalized in the same way as for the ghost vertex.

In the conventional discussion of the vertex renormalization, one considers such a vertex denoted by $\widetilde{\Lambda}^{a\mu}(p,q,k)$ that it is defined from $\Lambda^{a\mu}(p,q,k)$ by taking out a coupling constant. Obviously, the W-T identity obeyed by the $\widetilde{\Lambda}^{a\mu}(p,q,k)$ can be written out from (7.16) by taking away the coupling constant on the RHS of Eq. (7.16), that is

$$k_\mu \widetilde{\Lambda}^{a\mu}(p,q,k) = i\chi(k^2)[S_F^{-1}(p)\widetilde{\gamma}_2^a(p,q,k) - \widetilde{\gamma}_1^a(p,q,k) S_F^{-1}(q)]. \tag{7.27}$$

The renormalization of the vertex $\widetilde{\Lambda}^{a\mu}(p,q,k)$ usually is defined by

$$\widetilde{\Lambda}^{a\mu}(p,q,k) = Z_F^{-1} \widetilde{\Lambda}_R^{a\mu}(p,q,k) \tag{7.28}$$

where $Z_F$ is the quark-gluon vertex renormalization constant. When Eqs. (5.22), (7.18), (7.22) and (7.28) are inserted into Eq. (7.27) and then multiplying the both sides of Eq. (7.27) with a renormalized coupling constant, we arrive at

$$k_\mu \Lambda_R^{a\mu}(p,q,k) = i Z_F \widetilde{Z}_3 Z_2^{-1} Z_\gamma^{-1} g_R \chi_R(k^2)[S_F^{R-1}(p)\widetilde{\gamma}_{2R}^a(p,q,k) - \widetilde{\gamma}_{1R}^a(p,q,k) S_F^{R-1}(q)] \tag{7.29}$$

where

$$\Lambda_R^{a\mu}(p,q,k) = g_R \widetilde{\Lambda}_R^{a\mu}(p,q,k). \tag{7.30}$$

In comparison of Eq. (7.29) with Eq. (7.23) and considering the equality in Eq. (7.26), we find, the following identity must hold

$$\frac{Z_F}{Z_2} = \frac{\widetilde{Z}_1}{\widetilde{Z}_3}. \tag{7.31}$$

Combining the relations in Eqs. (5.28), (6.24) and (7.31), we have

$$\frac{Z_F}{Z_2} = \frac{\widetilde{Z}_1}{\widetilde{Z}_3} = \frac{Z_1}{Z_3} = \frac{Z_4}{Z_1}. \tag{7.32}$$

This just is the well-known S-T identity. This identity was obtained from the massless QCD and now, as has just been proved, it also holds for the massive QCD.

## VIII. EFFECTIVE COUPLING CONSTANT AND GLUON MASS

This section and the next section are used to perform one-loop renormalization of the massive QCD by using the renormalization group approach. As argued in our previous paper [30-32], when the renormalization is carried out in the mass-dependent momentum space subtraction scheme, the solutions to the RGEs satisfied by renormalized wave functions, propagators and vertices can be uniquely determined by the boundary conditions of the renormalized wave functions, propagators and vertices. In this case, an exact S-matrix element can be written in the form as given in the tree-diagram approximation provided that the coupling constant and particle masses in the matrix element are replaced by their effective (running) ones which are given by solving their renormalization group equations. Therefore, the task of renormalization is reduced to find the solutions of the RGEs for the renormalized coupling constant and particle masses. Suppose $F_R$ is a renormalized quantity. In the multiplicative renormalization, it is related to the unrenormalized one $F$ in such a way

$$F = Z_F F_R \tag{8.1}$$

where $Z_F$ is the renormalization constant of $F$. The $Z_F$ and $F_R$ are all functions of the renormalization point $\mu = \mu_0 e^t$ where $\mu_0$ is a fixed renormalization point corresponding the zero value of the group parameter $t$. Differentiating Eq. (8.1) with respect to $\mu$ and noticing that the $F$ is independent of $\mu$, we immediately obtain a renormalization group equation (RGE) satisfied by the function $F_R$ [21-23]

$$\mu \frac{dF_R}{d\mu} + \gamma_F F_R = 0 \tag{8.2}$$

where $\gamma_F$ is the anomalous dimension defined by

$$\gamma_F = \mu \frac{d}{d\mu} \ln Z_F. \tag{8.3}$$

Since the renormalization constant is dimensionless, the anomalous dimension can only depend on the ratio $\beta = \frac{m_R}{\mu}$ where $m_R$ denotes a, renormalized mass and $\gamma_F = \gamma_F(g_R, \beta)$ in which $g_R$ is the renormalized coupling constant and depends on $\mu$. Since the renormalization point is a momentum taken to subtract the divergence, we may set $\mu = \mu_0 \lambda$ where $\lambda = e^t$ which will be taken to be the same as in the scaling transformation of momentum $p = p_0 \lambda$. In the above, $\mu_0$ and $p_0$ are the fixed renormalization point and momentum respectively. When we set $F$ to be the coupling constant $g$ and noticing $\mu \frac{d}{d\mu} = \lambda \frac{d}{d\lambda}$, one can write from Eq. (8.2) the RGE for the renormalized coupling constant

$$\lambda \frac{dg_R(\lambda)}{d\lambda} + \gamma_g(\lambda) g_R(\lambda) = 0 \tag{8.4}$$

with

$$\gamma_g = \mu \frac{d}{d\mu} \ln Z_g. \tag{8.5}$$

According to the definition in Eq. (8.1) and the relation in Eq. (7.25), we may take,

$$Z_g = \frac{\widetilde{Z}_1}{\tilde{Z}_3 Z_3^{\frac{1}{2}}} \tag{8.6}$$

to calculate the anomalous dimension. As denoted in Eqs. (4.25) and (5.24), the renormalization constants $Z_3$, $\tilde{Z}_3$ and $\tilde{Z}_1$ are determined by the gluon self-energy, the ghost article self-energy and the ghost vertex correction, respectively. At one-loop level, the gluon self-energy is depicted in Figs. (1a)-(1d), the ghost article self-energy is shown in Fig. (2) and the ghost vertex correction is represented in Figs. (3a) and (3b). According to the Feynman rules which are the same as those for the massless QCD [16] except that the gluon propagator and the ghost particle one are now given in Eqs. (4.14) and (4.10), the expressions of the self-energies and the vertex correction are easily written out. For the gluon one-loop self-energy denoted by $-i\Pi_{\mu\nu}^{ab}(k)$, one can write

$$\Pi_{\mu\nu}^{ab}(k) = \sum_{i=1}^{4} \Pi_{\mu\nu}^{(i)ab}(k) \tag{8.7}$$

where $\Pi^{(1)ab}_{\mu\nu)}(k)$-$\Pi^{(4)ab}_{\mu\nu)}(k)$ represent the self-energies given in turn by Figs.(1a)-(1d). They are separately represented in the following:

$$\begin{aligned}\Pi^{(1)ab}_{\mu\nu}(k) = i\delta^{ab}\tfrac{3}{2}g^2\int\tfrac{d^4l}{(2\pi)^4}\tfrac{g^{\lambda\lambda'}g^{\rho\rho'}}{[l^2-M^2+i\varepsilon][(l+k)^2-M^2+i\varepsilon]}[g_{\mu\lambda}(l+2k)_\rho - g_{\lambda\rho}(2l+k)_\mu \\ +g_{\rho\mu}(l-k)_\lambda][g_{\nu\rho'}(l-k)_{\lambda'} - g_{\lambda'\rho'}(2l+k)_\nu + g_{\lambda'\nu}(l+2k)_{\rho'}],\end{aligned} \tag{8.8}$$

$$\Pi^{(2)ab}_{\mu\nu}(k) = -i\delta^{ab}3g^2\int\frac{d^4l}{(2\pi)^4}\frac{(l+k)_\mu l_\nu}{[(l+k)^2-M^2+i\varepsilon][l^2-M^2+i\varepsilon]}, \tag{8.9}$$

$$\Pi^{(3)ab}_{\mu\nu}(k) = -i\delta^{ab}3g^2\int\frac{d^4l}{(2\pi)^4}\frac{g^{\lambda\rho}}{(l^2-M^2+i\varepsilon)}(g_{\mu\nu}g_{\lambda\rho} - g_{\mu\rho}g_{\lambda\nu}) \tag{8.10}$$

and

$$\begin{aligned}\Pi^{(4)ab}_{\mu\nu}(k) = -i\delta^{ab}\tfrac{1}{2}g^2\int\tfrac{d^4l}{(2\pi)^4}\tfrac{1}{[(l-k)^2-m^2+i\varepsilon][l^2-m^2+i\varepsilon]} \\ \times Tr[\gamma_\mu(\mathbf{l}-\mathbf{k}+m)\gamma_\nu(\mathbf{l}+m)]\end{aligned} \tag{8.11}$$

where $\mathbf{l}=\gamma^\lambda l_\lambda$, $\mathbf{k}=\gamma^\lambda k_\lambda$. In the above, $f^{acd}f^{bcd} = 3\delta^{ab}$ and $Tr(T^aT^b) = \frac{1}{2}\delta^{ab}$ have been considered. It should be noted that in writing Eqs. (8.8)-(8.10), we choose to work in the Feynman gauge for simplicity. This choice is based on the fact that the massive QCD has been proved to be an unitary theory [18], that is to say, the S-matrix elements evaluated from the massive QCD are independent of gauge parameter. Therefore, we are allowed to choose a convenient gauge in the calculation. From Eqs. (8.8)-(8.11), it is clearly seen that

$$\Pi^{ab}_{\mu\nu}(k) = \delta^{ab}\Pi_{\mu\nu}(k) = \delta^{ab}\sum_{i=1}^{4}\Pi^{(i)}_{\mu\nu}(k). \tag{8.12}$$

By the dimensional regularization approach [33-37], the divergent integrals over $l$ in Eqs. (8.8)-(8.11) can be regularized in a $n$-dimensional space and easily calculated. The results are

$$\begin{aligned}\Pi^{(1)}_{\mu\nu}(k) = -\tfrac{3}{2}\tfrac{g^2}{(4\pi)^2}\int_0^1 dx\tfrac{1}{\varepsilon[k^2x(x-1)+M^2]^\varepsilon}\{g_{\mu\nu}[11x(x-1) \\ +5)k^2+9M^2] + 2[5x(x-1)-1]k_\mu k_\nu\},\end{aligned} \tag{8.13}$$

$$\begin{aligned}\Pi^{(2)}_{\mu\nu}(k) = \tfrac{3}{2}\tfrac{g^2}{(4\pi)^2}\int_0^1 dx\tfrac{1}{\varepsilon[k^2x(x-1)+M^2]^\varepsilon}\{[k^2x(x-1) \\ +M^2]g_{\mu\nu} + 2x(x-1)k_\mu k_\nu\},\end{aligned} \tag{8.14}$$

$$\Pi^{(3)}_{\mu\nu}(k) = \frac{9g^2}{(4\pi)^2}\frac{M^2}{\varepsilon}g_{\mu\nu} \tag{8.15}$$

and

$$\Pi^{(4)}_{\mu\nu}(k) = -\frac{4g^2}{(4\pi)^2}\int_0^1 dx\frac{k^2x(x-1)}{\varepsilon[k^2x(x-1)+m^2]^\varepsilon}[g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}] \tag{8.16}$$

where $\varepsilon = 2-\frac{n}{2}\to 0$ when $n\to 4$. In Eqs. (8.13)-(8.16), except for the $\varepsilon$ in the factor $1/\varepsilon[k^2x(x-1)+M^2]^\varepsilon$ and $1/\varepsilon[k^2x(x-1)+m^2]^\varepsilon$, we have set $\varepsilon\to 0$ in the other factors and terms by the consideration that this operation does not affect the calculated result of the anomalous dimension. According to the decomposition shown in Eqs. (4.15) and (4.16) and noticing $g_{\mu\nu} = \mathcal{P}^{\mu\nu}_T + \mathcal{P}^{\mu\nu}_L$, it is easy to get the transverse part of $\Pi_{\mu\nu}(k)$ from Eqs. (8.13)-(8.16) and furthermore, based on the decomposition denoted in Eq. (4.20), the functions $\Pi_1(k^2)$ and $\Pi_2(k^2)$ can be written out. The results are

$$\Pi_1(k^2) = -\frac{g^2}{(4\pi)^2}\int_0^1 dx\{\frac{15[2x(x-1)+1]}{2\varepsilon[k^2x(x-1)+M^2]^\varepsilon} + \sum_{i=1}^{N_f}\frac{4x(x-1)}{\varepsilon[k^2x(x-1)+m_i^2]^\varepsilon}\} \tag{8.17}$$

and

$$\Pi_2(k^2) = -\frac{g^2}{(4\pi)^2}\{\int_0^1 dx \frac{12}{\varepsilon[k^2x(x-1)+M^2]^\varepsilon} - \frac{27}{2\varepsilon M^{2\varepsilon}}\}. \tag{8.18}$$

It is clear that the both functions $\Pi_1(k^2)$ and $\Pi_2(k^2)$ are divergent in the four-dimensional space-time. When the divergences are subtracted in the mass-dependent momentum space subtraction scheme [34-37], in accordance with the definition in Eq. (4.25), we immediately obtain from the expression in Eq. (8.17) the one-loop renormalization constant $Z_3$ as follows

$$\begin{aligned} Z_3 &= 1 - \Pi_1(\mu^2) \\ &= 1 + \tfrac{g^2}{(4\pi)^2}\int_0^1 dx\{\tfrac{15[2x(x-1)+1]}{2\varepsilon[\mu^2x(x-1)+M^2]^\varepsilon} + \sum_{i=1}^{N_f}\tfrac{4x(x-1)}{\varepsilon[\mu^2x(x-1)+m_i^2]^\varepsilon}\}. \end{aligned} \tag{8.19}$$

Next, we turn to the ghost particle one-loop self-energy denoted by $-i\Omega^{ab}(q)$. From Fig. (2), in Feynman gauge, one can write

$$\Omega^{ab}(q) = i\delta^{ab}3g^2\int\frac{d^4l}{(2\pi)^4}\frac{q\cdot(q-l)}{[(q-l)^2-M^2+i\varepsilon][l^2-M^2+i\varepsilon]}. \tag{8.20}$$

By the dimensional regularization, it is easy to get

$$\Omega^{ab}(q) = \delta^{ab}q^2\hat{\Omega}(q^2) \tag{8.21}$$

where

$$\hat{\Omega}(q^2) = \frac{g^2}{(4\pi)^2}\int_0^1 dx\frac{3(x-1)}{\varepsilon[q^2x(x-1)+M^2]^\varepsilon}. \tag{8.22}$$

According to the definition given in Eq. (4.25) and the above expression , the one-loop renormalization constant of ghost particle propagator is of the form

$$\widetilde{Z}_3 = 1 - \hat{\Omega}(\mu^2) = 1 - \frac{g^2}{(4\pi)^2}\int_0^1 dx\frac{3(x-1)}{\varepsilon[\mu^2x(x-1)+M^2]^\varepsilon}. \tag{8.23}$$

Now, let us discuss the ghost vertex renormalization. In the one-loop approximation. the vertex defined by extracting out a coupling constant is expressed as

$$\widetilde{\Lambda}_\lambda^{abc}(p,q) = f^{abc}p_\lambda + \Lambda_{1\lambda}^{abc}(p,q) + \Lambda_{2\lambda}^{abc}(p,q) \tag{8.24}$$

where the first term is the bare vertex, the second and the third terms stand for the one-loop vertex corrections shown in Figs. (3a) and (3b) respectively. In the Feynman gauge, the vertex corrections are expressed as

$$\Lambda_{1\lambda}^{abc}(p,q) = -if^{abc}\frac{3}{2}g^2\int\frac{d^4l}{(2\pi)^4}\frac{p\cdot(q-l)(p-l)_\lambda}{[l^2-M^2+i\varepsilon][(p-l)^2-M^2+i\varepsilon][(q-l)^2-M^2+i\varepsilon]} \tag{8.25}$$

and

$$\Lambda_{2\lambda}^{abc}(p,q) = if^{abc}\frac{3}{2}g^2\int\frac{d^4l}{(2\pi)^4}\frac{l\cdot(p-q-l)p_\lambda - p\cdot lq_\lambda + p\cdot(2q-p+l)l_\lambda}{[l^2-M^2+i\varepsilon][(p-l)^2-M^2+i\varepsilon][(q-l)^2-M^2+i\varepsilon]} \tag{8.26}$$

where $f^{acd}f^{ebf}f^{dfc} = -\frac{3}{2}f^{abc}$ has been noted. By employing the dimensional regularization to compute the above integrals, it is not difficult to get

$$\Lambda_{1\lambda}^{abc}(p,q) = f^{abc}\frac{3}{2}\frac{g^2}{(4\pi)^2}\int_0^1 dx\int_0^1 dy\{\frac{\frac{1}{2}yp_\lambda}{\varepsilon\Theta_{xy}^\varepsilon} - \frac{1}{\Theta_{xy}}[p_\lambda A_1(p,q) + q_\lambda B_1(p,q)] - \frac{1}{8}p_\lambda\} \tag{8.27}$$

where

$$\begin{aligned} \Theta_{xy} &= p^2xy(xy-1) + q^2[(x-1)^2y + (x-1)]y - 2p\cdot qx(x-1)y^2 + M^2, \\ A_1(p,q) &= \{p\cdot q[1+(x-1)y] - p^2xy\}(1-xy)y, \\ B_1(p,q) &= \{p\cdot q[1+(x-1)y] - p^2xy\}(x-1)y^2 \end{aligned} \tag{8.28}$$

and

$$\Lambda_{2\lambda}^{abc}(p,q) = f^{abc}\frac{3}{2}\frac{g^2}{(4\pi)^2}\int_0^1 dx\int_0^1 dy\{\frac{\frac{3}{2}yp_\lambda}{\varepsilon\Theta_{xy}^\varepsilon} + \frac{1}{\Theta_{xy}}[p_\lambda A_2(p,q) + q_\lambda B_2(p,q)] - \frac{3}{8}p_\lambda\} \tag{8.29}$$

where

$$\begin{aligned} A_2(p,q) &= \{p^2(2xy - x^2y^2 - 1) - q^2[(x-1)y-1](x-1)y \\ &+p\cdot q[2-(3x-2)y+2x(x-1)y^2]\}y, \\ B_2(p,q) &= [p\cdot q(x-1) - p^2x]y^2. \end{aligned} \tag{8.30}$$

The divergences in the both vertices $\Lambda_{1\lambda}^{abc}(p,q)$ and $\Lambda_{2\lambda}^{abc}(p,q)$ may be subtracted at the renormalization point $p^2 = q^2 = \mu^2$ which implies $k = p - q = 0$, being consistent with the momentum conservation held at the vertices. Upon substituting Eqs. (8.27) and (8.29) in Eq. (8.24), at the renormalization point, one can get

$$\widetilde{\Lambda}_\lambda^{abc}(p,q)\mid_{p^2=q^2=\mu^2} = f^{abc}p_\lambda(1+\widetilde{L}_1) = \widetilde{Z}_1^{-1}f^{abc}p_\lambda \tag{8.31}$$

where

$$\widetilde{Z}_1 = 1 - \widetilde{L}_1 = 1 - \frac{3g^2}{(4\pi)^2}\int_0^1 dx\{\frac{x}{\varepsilon[\mu^2x(x-1)+M^2]^\varepsilon} - \frac{x^2(x-1)\mu^2}{\mu^2x(x-1)+M^2} - \frac{1}{4}\} \tag{8.32}$$

which is the one-loop renormalization constant of the ghost vertex.

Now we are ready to calculate the anomalous dimension $\gamma_g(\lambda)$. Substituting the expressions in Eqs. (8.6), (8.19), (8.23) and (8.32) into Eq. (8.5), it is easy to find an analytical expression of the anomalous dimension $\gamma_g(\lambda)$. When we set $\frac{M}{\mu} = \frac{\beta}{\lambda}$ and $\frac{m_i}{\mu} = \frac{\rho_i}{\lambda}$ with defining $\beta = \frac{M}{\Lambda}$ and $\rho_i = \frac{m_i}{\Lambda}$ (here we have set $\mu_0 \equiv \Lambda$), the expression of $\gamma_g(\lambda)$, in the approximation of order $g^2$, is given by

$$\gamma_g(\lambda) = \lim_{\varepsilon\to 0}[\mu\frac{d}{d\mu}\ln\widetilde{Z}_1 - \mu\frac{d}{d\mu}\ln\widetilde{Z}_3 - \frac{1}{2}\mu\frac{d}{d\mu}\ln Z_3] = \frac{g_R^2}{(4\pi)^2}F(\lambda) \tag{8.33}$$

where

$$\begin{aligned} F(\lambda) &= \tfrac{19}{2} - \tfrac{15\beta^2}{\lambda^2} + \tfrac{3\lambda^2}{2(\lambda^2-4\beta^2)} - (8 - \tfrac{10\beta^2}{\lambda^2} - \tfrac{\lambda^2}{\lambda^2-4\beta^2})\tfrac{3\beta^2}{\lambda\sqrt{\lambda^2-4\beta^2}} \\ &\times\ln\tfrac{\lambda-\sqrt{\lambda^2-4\beta^2}}{\lambda+\sqrt{\lambda^2-4\beta^2}} - \tfrac{2}{3}\sum_{i=1}^{N_f}[1 + \tfrac{6\rho_i^2}{\lambda^2} - \tfrac{12\rho_i^4}{\lambda^3\sqrt{\lambda^2-4\rho_i^2}}\ln\tfrac{\lambda-\sqrt{\lambda^2-4\rho_i^2}}{\lambda+\sqrt{\lambda^2-4\rho_i^2}}] \end{aligned} \tag{8.34}$$

in which $N_f$ denotes the number of quark flavors. We would like to note that the fixed renormalization point $\Lambda$ in $\beta$ and $\rho_i$ can be taken arbitrarily. For example, the $\Lambda$ may be chosen to be the mass of the quark of $N_f$ -th flavor. In this case, $\beta = M/m_{N_f}$ and $\rho_i = m_i/m_{N_f}$. In practice, the $\Lambda$ will be treated as a scaling parameter of renormalization.

With the $\gamma_g(\lambda)$ given above, the equation in Eq. (8.4) can be solved to give the effective coupling constant as follows

$$\alpha_R(\lambda) = \frac{\alpha_R}{1+\frac{\alpha_R}{2\pi}G(\lambda)} \tag{8.35}$$

where $\alpha_R(\lambda) = g_R^2(\lambda)/4\pi$, $\alpha_R = \alpha_R(1)$ and

$$G(\lambda) = \int_1^\lambda \frac{d\lambda}{\lambda}F(\lambda) = \varphi_1(\lambda) - \varphi_1(1) - \frac{1}{3}\sum_{i=1}^{N_f}[\varphi_2^i(\lambda) - \varphi_2^i(1)] \tag{8.36}$$

in which

$$\varphi_1(\lambda) = [(19 - \frac{10\beta^2}{\lambda^2})\frac{\sqrt{\lambda^2-4\beta^2}}{4\lambda} + \frac{3\lambda}{4\sqrt{\lambda^2-4\beta^2}}]\ln\frac{\lambda+\sqrt{\lambda^2-4\beta^2}}{\lambda-\sqrt{\lambda^2-4\beta^2}} + \frac{5\beta^2}{\lambda^2}, \tag{8.37}$$

$$\varphi_2^i(\lambda) = (1 + \frac{2\rho_i^2}{\lambda^2})\frac{\sqrt{\lambda^2-4\rho_i^2}}{\lambda}\ln\frac{\lambda+\sqrt{\lambda^2-4\rho_i^2}}{\lambda-\sqrt{\lambda^2-4\rho_i^2}} - \frac{4\rho_i^2}{\lambda^2} \tag{8.38}$$

and $\varphi_1(1) = \varphi_1(\lambda)\mid_{\lambda=1}$, $\varphi_2^i(1) = \varphi_2^i(\lambda)\mid_{\lambda=1}$. In the large momentum limit ($\lambda \to \infty$), we have

$$G(\lambda) = (11 - \frac{2}{3}N_f)\ln\lambda. \tag{8.39}$$

This just is the result for massless QCD which was obtained previously in the minimal subtraction scheme [38-40]. It should be noted that the expressions in Eqs. (8.34), (8.37) and (8.38) are obtained at the timelike subtraction point where the $\lambda$ is a real variable. We may also take spacelike momentum subtraction. For this kind of subtraction, corresponding to $\mu \to i\mu$, the variable $\lambda$ in Eqs. (8.34), (8.37) and (8.38) should be replaced by $i\lambda$ where $\lambda$ is still a real variable. It is easy to see that the function in Eq. (8.39) is the same for the both subtractions.

The behavior of the function $\alpha_R(\lambda)$ is graphically described in Figs. (4)-(6). Figs. (4) and (5) represent respectively the effective coupling constants obtained at the timelike subtraction point and the spacelike subtraction point, where we take the flavor $N_f = 3$ as an illustration. For comparison, we show in each of the figures three effective coupling constants which are obtained by the massive QCD, the massless QCD and the minimal subtraction, respectively. These effective coupling constants are respectively represented by the solid, dashed and dotted lines in the figures. To exhibit the dependence of the effective coupling constant on the number of quark flavors, in Fig. (6), we show three effective coupling constants given by the timelike momentum subtraction. The solid, dashed and dotted lines in the figure represent the effective coupling constants obtained by taking $N_f = 2, 3$ and 4, respectively. In our test, we find that the behavior of the $\alpha_R(\lambda)$ sensitively depends on the choice of the constant $\alpha_R$, the scaling parameter $\Lambda$, gluon mass $M_R$ and the quark masses $m_R$. Certainly, the parameters $\alpha_R$, $\Lambda$, $M_R$ and $m_R$ should be determined by fitting to experimental data. In our calculation, as an illustration, we take the quark masses to be constituent quark ones. The masses of up, down, strange and charm quarks are taken to be $m_u = m_d = 350MeV$, $m_s = 500MeV$ and $m_c = 1500MeV$. For the other parameters, we take $\alpha_R = 0.2$, $M_R = 600MeV$, $\Lambda = 500MeV$ in Figs. (4) and (5) and $\Lambda = 1500MeV$ in Fig. (6). From Fig. (4) it is seen that the effective coupling constant given by the massive QCD is an analytical function with a maximum at $\lambda = 1.346$, the effective coupling constant given by the massless QCD is also an analytical function with a peak around $\lambda = 1$, but the effective coupling constant given by the minimal subtraction has a singularity at $\lambda = 0.1746$. Fig. (5) indicates that the effective coupling constant given by the massive QCD (where gluon mass $M_R = 600MeV$), similar to the one given by the minimal subtraction, has a Landau pole at $\lambda = 0.1845$ which implies that the coupling constant is not applicable in the region $\lambda \leq 0.1845$. However, if the gluon mass is taken to be $M_R \leq 425.75MeV$, we find, the Landau pole disappears and the effective coupling constant, analogous to the effective coupling constant given by the massless QCD, becomes a smooth function in the whole region of momentum as illustrated by the dotted-dashed line in Fig. (5) which represents the effective coupling constant given by taking $M_R = 425.75MeV$. From Fig. (6) it is clear to see that in the low and intermediate region of momentum, the larger the flavor number $N_f$, the smaller the maximum of the effective coupling constant is and the positions of the maxima for different coupling constants are different from one another. This property of the effective coupling constants would give a notable effect on the theoretical hadron spectrum because for the calculation of hadron spectrum, the quarks in a hadron are assumed to move not too fast as suggested in the nonperturbative quark potential model, therefore, the behavior of the effective coupling constant in the low and intermediate domain of energy would play a dominate role In this case.

Let us proceed to derive the one-loop effective gluon mass. Setting $F_R = M_R$ in Eq. (8.2), we have the RGE for the renormalized gluon mass

$$\lambda\frac{dM_R(\lambda)}{d\lambda} + \gamma_M(\lambda)M_R(\lambda) = 0 \tag{8.40}$$

where

$$\gamma_M(\lambda) = \mu\frac{d}{d\mu}\ln Z_M. \tag{8.41}$$

From the last equality in Eq. (4.25) and Eqs. (8.17) and (8.18), in the approximation of order $g^2$, we can write

$$\begin{aligned} Z_M &= 1 + \tfrac{1}{2}[\Pi_1(\mu^2) + \Pi_2(\mu^2)] \\ &= 1 - \tfrac{g^2}{(4\pi)^2}\{\int_0^1 dx[\tfrac{3[10x(x-1)+13]}{4\varepsilon[k^2x(x-1)+M^2]^\varepsilon} + \sum_{i=1}^{N_f}\tfrac{2x(x-1)}{\varepsilon[k^2x(x-1)+m_i^2]^\varepsilon}] - \tfrac{27}{4\varepsilon M^{2\varepsilon}}\}. \end{aligned} \tag{8.42}$$

On inserting Eq. (8.42) into Eq. (8.41) and completing the differentiation with respect to $\mu$ and the integration over $x$, we find

$$
\begin{aligned}
\gamma_M(\lambda) = \tfrac{g_R^2}{(4\pi)^2}\{17 - \tfrac{15\beta^2}{\lambda^2} - \tfrac{3\beta^2(13\lambda^2-10\beta^2)}{\lambda^3\sqrt{\lambda^2-4\beta^2}}\ln\tfrac{\lambda-\sqrt{\lambda^2-4\beta^2}}{\lambda+\sqrt{\lambda^2-4\beta^2}} \\
-\tfrac{2}{3}\sum_{i=1}^{N_f}[1+\tfrac{6\rho_i^2}{\lambda^2} - \tfrac{12\rho_i^4}{\lambda^3\sqrt{\lambda^2-4\rho_i^2}}\ln\tfrac{\lambda-\sqrt{\lambda^2-4\rho_i^2}}{\lambda+\sqrt{\lambda^2-4\rho_i^2}}]\}.
\end{aligned}
\tag{8.43}
$$

With this anomalous dimension, the RGE in Eq. (8.40) can be solved to give an effective gluon mass such that

$$
M_R(\lambda) = M_R e^{-S_g(\lambda)} \tag{8.44}
$$

where $M_R = M_R(1)$ and

$$
S_g(\lambda) = \int_1^\lambda \frac{d\lambda}{\lambda}\gamma_M(\lambda). \tag{8.45}
$$

In general, the coupling constant $g_R$ in Eq. (8.43) may be taken to be the effective one. If the coupling constant is taken to be the constant $g_R$, the function $S_g(\lambda)$ can be explicitly represented as

$$
S_g(\lambda) = \frac{\alpha_R}{4\pi}\{\varphi_3(\lambda) - \varphi_3(1) - \frac{1}{3}\sum_{i=1}^{N_f}[\varphi_2^i(\lambda) - \varphi_2^i(1)]\} \tag{8.46}
$$

where

$$
\varphi_3(\lambda) = (17 - \frac{5\beta^2}{\lambda^2})\frac{\sqrt{\lambda^2-4\beta^2}}{2\lambda}\ln\frac{\lambda+\sqrt{\lambda^2-4\beta^2}}{\lambda-\sqrt{\lambda^2-4\beta^2}} + \frac{5\beta^2}{\lambda^2} \tag{8.47}
$$

and $\varphi_2^i(\lambda)$ was given in Eq. (8.38). In the large momentum limit,

$$
S_g(\lambda) = \frac{\alpha_R}{4\pi}(17 - \frac{2}{3}N_f)\ln\lambda. \tag{8.48}
$$

Therefore, we have

$$
\lim_{\lambda\to\infty} M_R(\lambda) = 0 \tag{8.49}
$$

which exhibits the asymptotically free behavior.

The behavior of the effective gluon mass $M_R(\lambda)$ may be discussed in the way similar to the discussion of the effective coupling constant. Here we only limit ourself to show in Fig. (7) the behavior of the effective gluon mass written in Eqs. (8.44)-(9.46) with taking $N_f = 3$ and the coupling constant being a constant. In Fig. (7), the solid line and the dashed line represent the effective gluon masses given by the timelike momentum subtraction and spacelike momentum subtraction, respectively. The solid line exhibits that for the timelike momentum, the effective gluon mass is an analytical function with a maximum at $\lambda = 1.233$. When $\lambda$ goes to infinity, the $M_R(\lambda)$ tends to zero rather rapidly, while, when $\lambda$ goes to zero, the $M_R(\lambda)$ abruptly falls to zero. The dashed line tells us that for the spacelike momentum, the $M_R(\lambda)$ keeps a constant $M_R$ in the region [0,1] of $\lambda$, while when $\lambda \to \infty$, the $M_R(\lambda)$ smoothly tends to zero.

## IX. EFFECTIVE QUARK MASS

Before deriving the one-loop effective quark mass, we need first to discuss the subtraction of the quark one-loop self-energy on the basis of the W-T identity represented in Eq. (7.16). For later convenience, the identity in Eq. (7.16) will be given in another form. Introducing new vertex functions $\hat{\Lambda}^{a\mu}(p,q)$ and $\hat{\gamma}_i^a(p,q)$ defined by

$$
\begin{aligned}
\Gamma^{a\mu}(p,q,k) = (2\pi)^4\delta^4(p-q+k)ig\hat{\Lambda}^{a\mu}(p,q) \\
\gamma_i^a(p,q,k) = -(2\pi)^4\delta^4(p-q+k)\hat{\gamma}_i^a(p,q)
\end{aligned}
\tag{9.1}
$$

where $i = 1,2$ and $\hat{\gamma}_i^a(p,q) = -\widetilde{\gamma}_i^a(p,q)$ and considering $k = q - p$, Eq. (7.16) can be rewritten as

$$
(p-q)_\mu\hat{\Lambda}^{a\mu}(p,q) = \chi(k^2)[S_F^{-1}(p)\hat{\gamma}_2^a(p,q) - \hat{\gamma}_1^a(p,q)S_F^{-1}(q)]. \tag{9.2}
$$

From the perturbative calculation, it can be found that In the lowest order, we have

$$\begin{aligned}\hat{\Lambda}_\mu^{(0)a}(p,q) &= \gamma_\mu T^a, \\ \hat{\gamma}_1^{(0)a}(p,q) = \hat{\gamma}_2^{(0)a}(p,q) &= T^a.\end{aligned} \tag{9.3}$$

In the one-loop approximation of order $g^2$, the quark-gluon vertex denoted by $\hat{\Lambda}_\mu^{(1)a}(p,q)$ is contributed from the two diagrams in Figs. (8a) and (8b) whose expressions can easily be written out. The quark-ghost vertex functions $\hat{\gamma}_i^{(1)a}(p,q)$ $(i=1,2)$ are contributed from Figs. (9a) and (9b) and can be represented as

$$\hat{\gamma}_i^{(1)a}(p,q) = T^a K_i(p,q) \tag{9.4}$$

where

$$K_1(p,q) = i\frac{3}{2}g^2 \int \frac{d^4l}{(2\pi)^4}\gamma^\mu S_F(l)(q-l)^\nu D_{\mu\nu}(p-l)\Delta(q-l) \tag{9.5}$$

and

$$K_2(p,q) = i\frac{3}{2}g^2 \int \frac{d^4l}{(2\pi)^4} S_F(l)\gamma^\mu D_{\mu\nu}(q-l)(p-l)^\nu \Delta(p-l). \tag{9.6}$$

It is clear that the above functions are logarithmically divergent. In the one-loop approximation, the function $\chi(k^2)$ can be written as $\chi(k^2) = 1 - \hat{\Omega}^{(1)}(k^2)$ where the ghost particle one-loop self-energy $\hat{\Omega}^{(1)}(k^2)$ was represented in Eq. (8.22). Thus, up to the order of $g^2$, with setting $\hat{\Lambda}_\mu^{(1)a}(p,q) = T^a\Lambda_\mu^{(1)}(p,q)$, we can write

$$\hat{\Lambda}_\mu^a(p,q) = T^a[\gamma_\mu + \Lambda_\mu^{(1)}(p,q)] \tag{9.7}$$

and

$$\chi(k^2)\hat{\gamma}_i^a(p,q) = T^a[1 + I_i(p,q)] \tag{9.8}$$

where

$$I_i(p,q) = K_i(p,q) - \Omega^{(1)}(k^2). \tag{9.9}$$

Upon substituting Eqs. (9.7) and (9.8) and the inverse of the quark propagator denoted in Eq. (7.17) into Eq. (9.2), then differentiating the both sides of Eq. (9.2) with respect to $p^\mu$ and finally setting $q = p$, in the order of $g^2$, we get

$$\overline{\Lambda}_\mu(p,p) = -\frac{\partial\Sigma(p)}{\partial p^\mu} \tag{9.10}$$

where

$$\begin{aligned}\overline{\Lambda}_\mu(p,p) = \Lambda_\mu^{(1)}(p,p) - \gamma_\mu I_2(p,p) - (\mathbf{p}-m)\tfrac{\partial I_2(p,q)}{\partial p^\mu} \mid_{q=p} \\ + \tfrac{\partial I_1(p,q)}{\partial p^\mu} \mid_{q=p} (\mathbf{p}-m)\end{aligned} \tag{9.11}$$

here $m$ is the $i$-th quark mass (hereafter the subscript of $m_i$ is suppressed for simplicity). It is emphasized that at one-loop level, the both sides of Eq. (9.11) are of the order of $g^2$. In the derivation of Eq. (9.11), the terms of orders higher than $g^2$ have been neglected. The identity in Eq. (9.10) formally is the same as we met in QED. By the subtraction at $\mathbf{p}=\mu$, the vertex $\overline{\Lambda}_\mu(p,p)$ will be expressed in the form

$$\overline{\Lambda}_\mu(p,p) = L\gamma_\mu + \overline{\Lambda}_\mu^c(p) \tag{9.12}$$

where $L$ is a divergent constant defined by

$$L = \overline{\Lambda}_\mu(p,p) \mid_{\mathbf{p}=\mu} \tag{9.13}$$

and $\overline{\Lambda}_\mu^c(p)$ is the finite part of $\overline{\Lambda}_\mu(p,p)$ satisfying the boundary condition

$$\overline{\Lambda}_\mu^c(p) \mid_{\mathbf{p}=\mu} = 0. \tag{9.14}$$

On integrating the identity in Eq. (9.10) over the momentum $p_\mu$ and considering the expression in Eq. (9.12), we obtain

$$\Sigma(p) = A + (\mathbf{p} - \mu)[B - C(p^2)] \tag{9.15}$$

where

$$A = \Sigma(\mu), \tag{9.16}$$

$$B = -L \tag{9.17}$$

and $C(p^2)$ is defined by

$$\int_{p_0^\mu}^{p^\mu} dp^\mu \overline{\Lambda}_\mu^c(p) = (\mathbf{p} - \mu)C(p^2). \tag{9.18}$$

Clearly, the expression in Eq. (9.15) gives the subtraction version for the quark self-energy which is required by the W-T identity and correct at least in the approximation of order $g^2$. With this subtraction, the quark propagator in Eq. (7.17) will be renormalized as

$$S_F(p) = \frac{Z_2}{\mathbf{p} - m_R - \Sigma_R(p)} \tag{9.19}$$

where $Z_2$ is the renormalization constant defined by

$$Z_2^{-1} = 1 - B, \tag{9.20}$$

$m_R$ is the renormalized quark mass defined as

$$m_R = Z_m^{-1} m \tag{9.21}$$

in which

$$Z_m^{-1} = 1 + Z_2[Am^{-1} + (1 - \mu m^{-1})B], \tag{9.22}$$

$Z_m$ is the quark mass renormalization constant and $\Sigma_R(p)$ is the finite correction of the self-energy satisfying the boundary condition $\Sigma_R(p)_{|p^2=\mu^2} = 0$.

Now we are in a position to discuss the one-loop renormalization of quark mass. The RGE for the renormalized quark mass can be written from Eq. (8.2) by setting $F = m$,

$$\lambda \frac{dm_R(\lambda)}{d\lambda} + \gamma_m(\lambda) m_R(\lambda) = 0 \tag{9.23}$$

where

$$\gamma_m(\lambda) = \mu \frac{d}{d\mu} \ln Z_m. \tag{9.24}$$

It is clear that to determine the one-loop renormalization constant $Z_m$, we first need to determine the divergent constants $A$ and $B$ from the self-energy represented in Eq. (9.15). The one-loop self-energy denoted by $-i\Sigma(p)$ can be written out from Fig. (10). In the Feynman gauge, it is

$$\Sigma(p) = -i\frac{4}{3}g^2 \int \frac{d^4k}{(2\pi)^4} \frac{\gamma^\mu(\mathbf{k} + \mathbf{p} + m)\gamma_\mu}{[(k+p)^2 - m^2 + i\varepsilon](k^2 - M^2 + i\varepsilon)} \tag{9.25}$$

where $\mathbf{k} = \gamma^\mu k_\mu$ and $\mathbf{p} = \gamma^\mu p_\mu$. By making use of the dimensional regularization to calculate the above integral, it is found that

$$\Sigma(p) = \frac{8}{3}\frac{g^2}{(4\pi)^2} \int_0^1 dx \frac{(x-1)\mathbf{p} + 2m}{\varepsilon[p^2 x(x-1) + m^2 x + M^2(1-x)]^\varepsilon}. \tag{9.26}$$

According to Eq. (9.16), we have

$$A=\Sigma(p)\mid_{\mathbf{p}=\mu}=\frac{8}{3}\frac{g^2}{(4\pi)^2}\int_0^1 dx\frac{(x-1)\mu+2m}{\varepsilon[\mu^2x(x-1)+m^2x+M^2(1-x)]^{\varepsilon}}. \tag{9.27}$$

With the aid of the following formula

$$\frac{1}{a^{\varepsilon}}-\frac{1}{b^{\varepsilon}}=\varepsilon\int_0^1 dx\frac{b-a}{[ax+b(1-x)]^{1+\varepsilon}}, \tag{9.28}$$

one can get from Eqs. (9.15) and (9.27) that

$$\begin{array}{c} B=[\Sigma(p)-A](\mathbf{p}-m)^{-1}\mid_{\mathbf{p}=\mu} \\ =\frac{8}{3}\frac{g^2}{(4\pi)^2}\int_0^1 dx\{\frac{(x-1)}{\varepsilon[\mu^2x(x-1)+m^2x+M^2(1-x)]^{\varepsilon}}-\frac{2x(x-1)[(x-1)\mu^2+2m\mu]}{\mu^2x(x-1)+m^2x+M^2(1-x)}\} \end{array} \tag{9.29}$$

where $C(\mu^2)=0$ has been considered. On inserting Eqs. (9.27) and (9.29) into Eq. (9.22) and noting that in the approximation of order $g^2$, $Z_2\simeq 1$ should be taken in Eq. (9.22), it can be found that

$$\begin{array}{c} Z_m=1-\frac{A}{m}-(1-\frac{\mu}{m})B \\ =1-\frac{g^2}{(4\pi)^2}\frac{8}{3}\int_0^1 dx\{\frac{(x+1)}{\varepsilon[\mu^2x(x-1)+m^2x+M^2(1-x)]^{\varepsilon}}+\frac{2x(x-1)[(x-1)(\mu/m-1)\mu^2+2(\mu^2-m\mu]}{\mu^2x(x-1)+m^2x+M^2(1-x)}\}. \end{array} \tag{9.30}$$

When substituting Eq. (9.30) in Eq. (9.24) and applying the familiar integration formulas, through a lengthy calculation, we obtain

$$\begin{array}{c} \gamma_m(\lambda)=\frac{4\alpha_R}{3\pi}\{\xi_1(\lambda)+\xi_2(\lambda)\ln\frac{\beta}{\rho}+\frac{2}{\lambda^2K(\lambda)}\xi_3(\lambda) \\ +\frac{1}{2\lambda^4}[\xi_4(\lambda)+\frac{4}{K(\lambda)}\xi_5(\lambda)]J(\lambda)\} \end{array} \tag{9.31}$$

where

$$\xi_1(\lambda)=\frac{1}{\lambda^2}[\frac{\lambda^3}{2\rho}+\frac{3}{2}\lambda^2+(\beta^2-3\rho^2)\frac{\lambda}{\rho}+\rho^2-\beta^2], \tag{9.32}$$

$$\begin{array}{c} \xi_2(\lambda)=\frac{1}{\lambda^4}[\frac{\beta^2}{\rho}\lambda^3+(6\rho^2-7\beta^2)\lambda^2 \\ -\frac{3}{\rho}(\rho^2-\beta^2)(3\rho^2-\beta^2)\lambda+3(\rho^2-\beta^2)^2], \end{array} \tag{9.33}$$

$$\begin{array}{c} \xi_3(\lambda)=\frac{\beta^2}{\rho}\lambda^5-(2\rho^2+3\beta^2)\lambda^4+\frac{1}{\rho}(3\rho^4+3\rho^2\beta^2-2\beta^4)\lambda^3 \\ +(\rho^2-\beta^2)(\rho^2-4\beta^2)\lambda^2-\frac{1}{\rho}(\rho^2-\beta^2)(3\rho^4-4\rho^2\beta^2+\beta^4)\lambda+(\rho^2-\beta^2)^3, \end{array} \tag{9.34}$$

$$\begin{array}{c} \xi_4(\lambda)=\frac{\beta^2}{\rho}\lambda^5-(6\rho^2+7\beta^2)\lambda^4+\frac{1}{\rho}(9\rho^4+11\rho^2\beta^2-8\beta^4)\lambda^3 \\ +11(\rho^2-\beta^2)(\rho^2-2\beta^2)\lambda^2-\frac{7}{\rho}(\rho^2-\beta^2)^2(3\rho^2-\beta^2)\lambda+7(\rho^2-\beta^2)^3, \end{array} \tag{9.35}$$

$$\begin{array}{c} \xi_5(\lambda)=\frac{\beta^4}{\rho}\lambda^7-(2\rho^4+3\beta^4)\lambda^6+\frac{1}{\rho}(3\rho^6+4\rho^2\beta^4-3\beta^6)\lambda^5 \\ +(\rho^2-\beta^2)(3\rho^4-\rho^2\beta^2-7\beta^4)\lambda^4-\frac{1}{\rho}(\rho^2-\beta^2)^2(6\rho^4+5\rho^2\beta^2-3\beta^4)\lambda^3 \\ +5\beta^2(\rho^2-\beta^2)^3\lambda^2+\frac{1}{\rho}(\rho^2-\beta^2)^4(3\rho^2-\beta^2)\lambda-(\rho^2-\beta^2)^5, \end{array} \tag{9.36}$$

$$K(\lambda)=\lambda^4-2(\beta^2+\rho^2)\lambda^2+(\rho^2-\beta^2)^2 \tag{9.37}$$

and

$$\begin{array}{c} \gamma_m(\lambda)=\frac{4\alpha_R}{3\pi}\{\xi_1(\lambda)+\xi_2(\lambda)\ln\frac{\beta}{\rho}+\frac{2}{\lambda^2K(\lambda)}\xi_3(\lambda) \\ +\frac{1}{2\lambda^4}[\xi_4(\lambda)+\frac{4}{K(\lambda)}\xi_5(\lambda)]J(\lambda)\} \end{array} \tag{9.31}$$

where

$$\xi_1(\lambda) = \frac{1}{\lambda^2}[\frac{\lambda^3}{2\rho} + \frac{3}{2}\lambda^2 + (\beta^2 - 3\rho^2)\frac{\lambda}{\rho} + \rho^2 - \beta^2], \tag{9.32}$$

$$\begin{gathered} \xi_2(\lambda) = \tfrac{1}{\lambda^4}[\tfrac{\beta^2}{\rho}\lambda^3 + (6\rho^2 - 7\beta^2)\lambda^2 \\ -\tfrac{3}{\rho}(\rho^2 - \beta^2)(3\rho^2 - \beta^2)\lambda + 3(\rho^2 - \beta^2)^2], \end{gathered} \tag{9.33}$$

$$\begin{gathered} \xi_3(\lambda) = \tfrac{\beta^2}{\rho}\lambda^5 - (2\rho^2 + 3\beta^2)\lambda^4 + \tfrac{1}{\rho}(3\rho^4 + 3\rho^2\beta^2 - 2\beta^4)\lambda^3 \\ +(\rho^2 - \beta^2)(\rho^2 - 4\beta^2)\lambda^2 - \tfrac{1}{\rho}(\rho^2 - \beta^2)(3\rho^4 - 4\rho^2\beta^2 + \beta^4)\lambda + (\rho^2 - \beta^2)^3, \end{gathered} \tag{9.34}$$

$$\begin{gathered} \xi_4(\lambda) = \tfrac{\beta^2}{\rho}\lambda^5 - (6\rho^2 + 7\beta^2)\lambda^4 + \tfrac{1}{\rho}(9\rho^4 + 11\rho^2\beta^2 - 8\beta^4)\lambda^3 \\ +11(\rho^2 - \beta^2)(\rho^2 - 2\beta^2)\lambda^2 - \tfrac{7}{\rho}(\rho^2 - \beta^2)^2(3\rho^2 - \beta^2)\lambda + 7(\rho^2 - \beta^2)^3, \end{gathered} \tag{9.35}$$

$$\begin{gathered} \xi_5(\lambda) = \tfrac{\beta^4}{\rho}\lambda^7 - (2\rho^4 + 3\beta^4)\lambda^6 + \tfrac{1}{\rho}(3\rho^6 + 4\rho^2\beta^4 - 3\beta^6)\lambda^5 \\ +(\rho^2 - \beta^2)(3\rho^4 - \rho^2\beta^2 - 7\beta^4)\lambda^4 - \tfrac{1}{\rho}(\rho^2 - \beta^2)^2(6\rho^4 + 5\rho^2\beta^2 - 3\beta^4)\lambda^3 \\ +5\beta^2(\rho^2 - \beta^2)^3\lambda^2 + \tfrac{1}{\rho}(\rho^2 - \beta^2)^4(3\rho^2 - \beta^2)\lambda - (\rho^2 - \beta^2)^5, \end{gathered} \tag{9.36}$$

$$K(\lambda) = \lambda^4 - 2(\beta^2 + \rho^2)\lambda^2 + (\rho^2 - \beta^2)^2 \tag{9.37}$$

and

$$\begin{gathered} \gamma_m(\lambda) = \tfrac{4\alpha_R}{3\pi}\{\xi_1(\lambda) + \xi_2(\lambda)\ln\tfrac{\beta}{\rho} + \tfrac{2}{\lambda^2 K(\lambda)}\xi_3(\lambda) \\ +\tfrac{1}{2\lambda^4}[\xi_4(\lambda) + \tfrac{4}{K(\lambda)}\xi_5(\lambda)]J(\lambda)\} \end{gathered} \tag{9.31}$$

where


$$\xi_1(\lambda) = \frac{1}{\lambda^2}[\frac{\lambda^3}{2\rho} + \frac{3}{2}\lambda^2 + (\beta^2 - 3\rho^2)\frac{\lambda}{\rho} + \rho^2 - \beta^2], \tag{9.32}$$

$$\begin{gathered} \xi_2(\lambda) = \tfrac{1}{\lambda^4}[\tfrac{\beta^2}{\rho}\lambda^3 + (6\rho^2 - 7\beta^2)\lambda^2 \\ -\tfrac{3}{\rho}(\rho^2 - \beta^2)(3\rho^2 - \beta^2)\lambda + 3(\rho^2 - \beta^2)^2], \end{gathered} \tag{9.33}$$

$$\begin{gathered} \xi_3(\lambda) = \tfrac{\beta^2}{\rho}\lambda^5 - (2\rho^2 + 3\beta^2)\lambda^4 + \tfrac{1}{\rho}(3\rho^4 + 3\rho^2\beta^2 - 2\beta^4)\lambda^3 \\ +(\rho^2 - \beta^2)(\rho^2 - 4\beta^2)\lambda^2 - \tfrac{1}{\rho}(\rho^2 - \beta^2)(3\rho^4 - 4\rho^2\beta^2 + \beta^4)\lambda + (\rho^2 - \beta^2)^3, \end{gathered} \tag{9.34}$$

$$\begin{gathered} \xi_4(\lambda) = \tfrac{\beta^2}{\rho}\lambda^5 - (6\rho^2 + 7\beta^2)\lambda^4 + \tfrac{1}{\rho}(9\rho^4 + 11\rho^2\beta^2 - 8\beta^4)\lambda^3 \\ +11(\rho^2 - \beta^2)(\rho^2 - 2\beta^2)\lambda^2 - \tfrac{7}{\rho}(\rho^2 - \beta^2)^2(3\rho^2 - \beta^2)\lambda + 7(\rho^2 - \beta^2)^3, \end{gathered} \tag{9.35}$$

$$\begin{gathered} \xi_5(\lambda) = \tfrac{\beta^4}{\rho}\lambda^7 - (2\rho^4 + 3\beta^4)\lambda^6 + \tfrac{1}{\rho}(3\rho^6 + 4\rho^2\beta^4 - 3\beta^6)\lambda^5 \\ +(\rho^2 - \beta^2)(3\rho^4 - \rho^2\beta^2 - 7\beta^4)\lambda^4 - \tfrac{1}{\rho}(\rho^2 - \beta^2)^2(6\rho^4 + 5\rho^2\beta^2 - 3\beta^4)\lambda^3 \\ +5\beta^2(\rho^2 - \beta^2)^3\lambda^2 + \tfrac{1}{\rho}(\rho^2 - \beta^2)^4(3\rho^2 - \beta^2)\lambda - (\rho^2 - \beta^2)^5, \end{gathered} \tag{9.36}$$

$$K(\lambda) = \lambda^4 - 2(\beta^2 + \rho^2)\lambda^2 + (\rho^2 - \beta^2)^2 \tag{9.37}$$

and


$$J(\lambda) = \frac{1}{\sqrt{K(\lambda)}}\ln\frac{\lambda^2 - (\beta^2 + \rho^2) - \sqrt{K(\lambda)}}{\lambda^2 - (\beta^2 + \rho^2) + \sqrt{K(\lambda)}}. \tag{9.38}$$

With the anomalous dimension given above, the equation in Eq. (9.23) can be solved and gives the effective mass for a quark as follows

$$m_R(\lambda) = m_R e^{-S_q(\lambda)} \tag{9.39}$$

where

$$S_q(\lambda) = \int_1^\lambda \frac{d\lambda}{\lambda}\gamma_m(\lambda). \tag{9.40}$$

This integral is not able to be analytically calculated even though the coupling constant in Eq. (9.31) is taken to be a constant. In the large momentum limit ($\lambda \to \infty$), Eq. (9.31) tends to

$$\gamma_m(\lambda) \approx \frac{2\alpha_R}{3\pi\rho}\lambda. \tag{9.41}$$

In this limit, it is seen that in the time-like momentum space, we have

$$m_R(\lambda) \approx m_R e^{-\frac{2\alpha_R}{3\pi\rho}\lambda} \to 0. \tag{9.42}$$

Graphically, we only show in Fig. (11) the $s$ quark effective mass $m_R(\lambda)$ given by the timelike momentum subtraction with the coupling constant being taken to be a constant. For other quarks, the behavior of their effective masses is similar. In Fig. (11), the solid line represents the effective mass given by the massive QCD, while the dashed line represents the effective mass given by the massless QCD. From the figure, we see that the effective mass with a finite gluon mass behaves as a constant equal to the mass $m_R$ in the region [0,1] of $\lambda$ and then rapidly falls to zero when $\lambda$ goes from unit to infinity. The effective mass with zero-gluon mass has a peak around $\lambda = 1.38$. The appreciable difference between the both effective masses occurs in the region [0,10] of $\lambda$. The effective quark mass given by the spacelike momentum subtraction can directly be written out from Eqs. (9.31)-(9.40) by replacing the $\lambda$ in $\gamma_m(\lambda)$ with $i\lambda$ and hence the $m_R(\lambda)$ becomes a complex function. By numerical calculations, it is found that either the real part or the imaginary part of the $m_R(\lambda)$ behaves as an oscillating function with a damping amplitude as exhibited in Fig. (12). In the figure, the solid and the dashed lines represent respectively the real part and imaginary part of the effective quark mass which is given by the massive QCD, while the dotted line shows the real part of the effective quark mass which is given by the massless QCD. It is noted that in the most of practical applications to the both of scattering and bound state problems, only the effective quark mass given by the timelike momentum subtraction is concerned .

## X. CONCLUSIONS AND DISCUSSIONS

In this paper, it has been shown that as the massless QCD, the massive QCD established on the basis of gauge-invariance has a set of BRST-transformations under which the effective action and generating functional are invariant. From the BRST-invariance, we derived a set of W-T identities satisfied by the generating functionals for full Green functions, connected Green functions and proper vertex functions. Furthermore, from the above identities, we derived the W-T identities respected by the gluon propagator, the three-line and four-line proper gluon vertices and the quark-gluon proper vertex. Based on these identities we discussed the renormalization of the propagators and the vertices. In particular, from the renormalized forms of the W-T identities obeyed by propagators and vertices, the S-T identity for the renormalization constants is naturally deduced. This identity is helpful for the renormalization by means of the renormalization group approach. To show the renormalizability of the massive QCD, the one-loop renormalization is performed by the renormalization group method. In this renormalization, the analytical expressions of the one-loop effective coupling constant, gluon mass and quark mass have been derived. Since the renormalization was carried out by employing the mass-dependent momentum space subtraction scheme and exactly respecting the W-T identities, the results obtained are faithful and allow us to discuss the physical behaviors of the effective coupling constant and masses in the whole range of momentum (or distance). Particularly, the previous result given in the minimal subtraction scheme for massless QCD is naturally recovered in the large momentum limit.

As shown in sections 8 and 9, in the mass-dependent renormalization, it is necessary to distinguish the results given by the timelike momentum subtraction from the corresponding ones obtained by the spacelike momentum subtraction. For example, one can see from Figs. (4) and (5) that the effective coupling constants given in the timelike and spacelike subtraction schemes have different behaviors in the low and intermediate energy region although in the large momentum limit, the difference between the both coupling constants disappears. Obviously, the both

results obtained in the timelike and spacelike momentum subtraction schemes are meaningful and suitable for different physical processes. For instance, when we study the quark-quark scattering taking place in the t-channel, the transfer momentum in the gluon propagator is spacelike. In this case, it is suitable to take the effective coupling constant and gluon mass given by the spacelike momentum subtraction. If we investigate the quark-antiquark annihilation process which takes place in the s-channel, since the transfer momentum is timelike, the effective coupling constant and gluon mass given in the timelike momentum subtraction scheme should be used. It is also seen from sections 8 and 9 that the gluon mass gives a considerable effect on the behaviors of the effective coupling constant and particle masses. In particular, the gluon mass plays an crucial role to determine the singular or analytical behavior of the effective coupling constant. Since the gauge-invariance does not exclude the gluon to have a mass, it is interesting to examine the gluon mass effect on physical processes. At present, the massless QCD has widely been recognized to be the candidate of the strong interaction theory and has been proved to be compatible with the present high energy experiments. However, we think, the massive QCD would be more favorable to explain the strong interaction phenomenon, particularly, at the low energy region because the massive gluon would make the force range more shorter than that caused by the massless gluon. As for the high energy and large momentum transfer phenomena, as seen from the massive gluon propagator, the gluon mass gives little influence on the theoretical result in this case so that the massive QCD could not conflict with the well-established results gained from the massless QCD in the high energy domain.

At last, we would like to make some remarks on the nilpotency problem of the BRST-external sources. In comparison of the BRST-transformations and the W-T identities for the massive QCD with those for the massless QCD, it is seen that they formally are almost the same. The only difference is that the BRST-transformation for the ghost particle field $C^a(x)$ written in Eq. (2.25) has an extra term proportional to $\sigma^2$, the ghost particle mass squared. Due to this term, there also appear some $\sigma^2-$dependent terms in the W-T identities as denoted in Eqs. (3.18), (4.2), (4.8), (5.11) and (6.18). But, in the physical Landau gauge where $\sigma = 0$, all the $\sigma^2$-dependent terms disappear. In this case, all the BRST-transformations and W-T identities for the massive QCD are formally identical to those for the massless QCD. As one knows, for the massless QCD, the composite field functions $\Delta\Phi_i$ as defined in Eq. (3.3) have the nilpotency property: $\delta\Delta\Phi_i = 0$ under the BRST-transformations [13-16] which guarantee the BRST-invariance of the BRST-source terms introduced in the generating functional. This nilpotency property is still preserved for the massive QCD established in the Landau gauge because in the Landau gauge, the BRST-transformations are identical to those for the massless QCD. However, for the massive QCD set up in arbitrary gauges, we find $\delta\Delta\Phi_i \neq 0$, the nilpotency loses due to nonzero of the ghost particle spurious mass $\sigma$. In this case, as pointed out in section 3, to ensure the BRST-invariance of the source terms, we may simply require the sources $u_i$ to satisfy the condition denoted in Eq. (3.7). The definition in Eq. (3.7) for the sources is reasonable. Why say so? Firstly, we note that the original W-T identity formulated in Eq. (3.2) does not involve the BRST- sources. This identity is suitable to use in practical applications. Introduction of the BRST source terms in the generating functional is only for the purpose of representing the identity in Eq. (3.2) in a convenient form, namely, to represent the composite field functions in the identity in terms of the differentials of the generating functional with respect to the corresponding sources. For this purpose, we may start from the generating functional defined in Eq. (3.4) to re-derive the identity in Eq. (3.2). In doing this, it is necessary to require the source terms $u_i\triangle\Phi_i$ to be BRST-invariant so as to make the derived identity coincide with that given in Eq. (3.2). How to ensure the source terms to be BRST-invariant? If the composite field functions $\triangle\Phi_i$ are nilpotent under the BRST-transformation, the BRST-invariance of the source terms is certainly guaranteed. Nevertheless, the nilpotency of the functions $\triangle\Phi_i$ is not a uniquely necessary condition to ensure the BRST- invariance of the source terms, particularly, in the case where the functions $\triangle\Phi_i$ are not nilpotent. In the latter case, considering that under the BRST- transformations, the functions $\triangle\Phi_i$ can be, in general, expressed as $\delta\Delta\Phi_i = \xi\widetilde{\Phi}_i$ where the $\widetilde{\Phi}_i$ are some nonvanishing functions, we may alternatively require the sources $u_i$ to satisfy the condition shown in Eq. (3.7) so as to guarantee the source terms to be BRST- invariant. Actually, this is a general trick to make the source terms to be BRST-invariant in spite of whether the functions $\triangle\Phi_i$ are nilpotent or not. As mentioned before, the sources themselves have no physical meaning. They are, as a mathematical tool, introduced into the generating functional just for performing the differentiations. For this purpose, only a certain algebraic and analytical properties of the sources are necessarily required. Particularly, in the differentiations, only the infinitesimal property of the sources are concerned. Therefore, the sources defined in Eq. (3.7) are mathematically suitable for the purpose of introducing them. The reasonability of the arguments stated above for the source terms is substantiated by the correctness of the W-T identities derived in sections 4-7. Even though the identities in Eqs. (4.1) and (4.2) are derived from the W-T identity in Eq. (3.8) which is represented in terms of the differentials with respect to the BRST- sources, they give rise to correct relations between the propagators and/or vertices. For example, the correctness of the relation in Eq. (4.8) can easily be verified by the free propagators written in Eqs. (4.10) and (4 14). These propagators are usually derived from the generating functional in Eq. (2.12) by employing the perturbation method without concerning the BRST-source terms and the nilpotency of the BRST- transformations. A powerful argument of proving the correctness of the way of introducing the BRST-sources is that after completing the differentiations

in Eq. (3.8) and setting the BRST-sources to vanish, we immediately obtain the W-T identity in Eq. (3.2) which is irrelevant to the BRST-sources. Therefore, all identities or relations derived from the W-T identity in Eq. (3.8) are completely the same as those derived from the identity in Eq. (3.2). An important example of showing this point will be presented in Appendix where an identity derived from the W-T identities in Eqs. (3.8) can equally be derived from the generating functional in Eq. (2.12) which does not involve the BRST-sources.

## XI. ACKNOWLEDGMENT

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## XII. APPENDIX: ALTERNATIVE DERIVATION OF THE W-T IDENTITY

In this appendix, we give an alternative derivation of a W-T identity without concerning the nilpotency of the composite field functions appearing in the BRST-source terms and show that the result is equal to the one obtained from . generating functional containing the BRST-external sources. Let us start from the generating functional of Green functions given in Eqs. (2.12) and (2.13). For simplicity of statement, we omit the fermion field functions in the generating functional and rewrite the functional in the form

$$\begin{array}{c} Z[J,\overline{\xi},\xi]=\frac{1}{N}\int\mathcal{D}[A.\bar{C}.C]exp\{iS+i\int d^4x[-\frac{1}{2\alpha}(\partial^\mu A^a_\mu)^2+J^{a\mu}A^a_\mu \\ +\overline{\xi}^aC^a+\bar{C}^a\xi^a]+i\int d^4xd^4y\bar{C}^a(x)M^{ab}(x,y)C^b(y)\} \end{array} \tag{A1}$$

where

$$S=\int d^4x[-\frac{1}{4}F^{a\mu\nu}F^a_{\mu\nu}+\frac{1}{2}M^2A^{a\mu}A^a_\mu] \tag{A2}$$

and

$$M^{ab}(x,y)=\partial^\mu_x[\mathcal{D}^{ab}_\mu(x)\delta^4(x-y)] \tag{A3}$$

in which $\mathcal{D}^{ab}_\mu(x)$ was defined in Eq. (2.9).

When we make the following translation transformations in Eq. (A1)

$$\begin{array}{c} C^a(x)\rightarrow C^a(x)-\int d^4y(M^{-1})^{ab}(x,y)K^b(y) \\ \bar{C}^a(x)\rightarrow\bar{C}^a(x)-\int d^4y\bar{K}^b(y)(M^{-1})^{ba}(y,x) \end{array} \tag{A4}$$

and complete the integration over the ghost field variables, Eq. (A1) will be expressed as

$$Z[J,\bar{K},K]=e^{-i\int d^4xd^4y\overline{\xi}^a(x)(M^{-1})^{ab}(x,y,\delta/i\delta J)\xi^b(y)}Z[J] \tag{A5}$$

where $Z[J]$ is the generating functional without the external sources of ghost fields [13,17]

$$Z[J]=\frac{1}{N}\int\mathcal{D}(A)\Delta_F[A]exp\{iS+i\int d^4x[-\frac{1}{2\alpha}(\partial^\mu A^a_\mu)^2+J^{a\mu}A^a_\mu]\} \tag{A6}$$

in which

$$\Delta_F[A]=detM[A] \tag{A7}$$

here the matrix $M[A]$ was defined in Eq. (A3). From Eq. (A5), we may obtain the ghost particle propagator in the presence of the external source $J$

$$\begin{array}{c} i\Delta^{ab}[x,y,J]=\frac{\delta^2Z[J,\bar{K},K]}{\delta\overline{\xi}^a(x)\delta\xi^b(y)}|_{\overline{K}=K=0} \\ =i(M^{-1})_{ab}[x,y,\frac{\delta}{i\delta J}]Z[J]. \end{array} \tag{A8}$$

The above result allows us to rewrite the W-T identity in Eq. (4.1) in terms of the generating functional $Z[J]$ when completing the derivative with respect to $u^{b\nu}(y)$ and setting $\overline{\xi}^a(x)=\xi^b(y)=0$,

$$\frac{1}{\alpha}\partial_x^\mu \frac{\delta Z[J]}{i\delta J^{a\mu}(x)} - \int d^4y J^{b\mu}(y) D_\mu^{bd}[y, \frac{\delta}{i\delta J}](M^{-1})^{da}(y, x, \frac{\delta}{i\delta J}) Z[J] = 0 \tag{A9}$$

where

$$D_\mu^{bd}(y) = \mathcal{D}_\mu^{bd}(y) - \frac{\sigma^2}{\Box_y}\partial_\mu^y \delta^{bd} \tag{A10}$$

is the ordinary covariant derivative. On completing the differentiations with respect to the source $J$, Eq. (A9) reads

$$\begin{gathered} \frac{1}{N}\int \mathcal{D}[A]\Delta_F[A] exp\{iS + i\int d^4x[-\frac{1}{2\alpha}(\partial^\mu A_\mu^a)^2 + J^{a\mu}A_\mu^a]\} \\ \times[\int d^4y J^{b\mu}(y) D_\mu^{bc}(y)(M^{-1})^{ca}(y,x) - \frac{1}{\alpha}\partial^\nu A_\nu^a(x)] \\ = 0 \end{gathered} \tag{A11}$$

By making use of Eqs. (A3), (A8) and (A10), the ghost equation shown in Eq. (4.2) may be written as

$$M^{ac}[x, \frac{\delta}{i\delta J}](M^{-1})^{cb}[x, y, \frac{\delta}{i\delta J}]Z[J] = \delta^{ab}\delta^4(x-y)Z[J] \tag{A12}$$

When the source $J$ is turned off, we get the equation written in Eq. (2.24) which affirms the fact that the ghost particle propagator is just the inverse of the matrix $M$.

To confirm the correctness of the identity given in Eq. (A11), we derive the identity newly by starting from the generating functional written in Eq. (A6) which does not involve the BRST-external sources. Let us make the ordinary gauge transformation $\delta A_\mu^a = D_\mu^{ab}\theta^b$ to the generating functional in Eq. (A6). Considering the gauge-invariance of the functional integral, the integration measure and the functional $\triangle_F[A] = \det M[A]$, we get [13,17]

$$\begin{gathered} \delta Z[J] = \frac{1}{N}\int D(A)\triangle_F[A]\int d^4y[J^{b\mu}(y) + M^2A^{b\mu}(y) \\ -\frac{1}{\alpha}\partial^\nu A_\nu^b \partial_y^\mu] D_\mu^{bc}(y)\theta^c(y)\exp\{iS + i\int d^4x[-\frac{1}{2\alpha}(\partial^\mu A_\mu^a)^2 + J^{a\mu}A_\mu^a]\} \\ = 0 \end{gathered} \tag{A13}$$

According to the well-known procedure, the group parameter $\theta^a(x)$ in Eq. (A13) may be determined by the following equation [10,14,17]

$$M^{ab}(x)\theta^b(x) \equiv \partial_x^\mu(\mathcal{D}_\mu^{ab}(x)\theta^b(x)) = \lambda^a(x) \tag{A14}$$

where $\lambda^a(x)$ is an arbitrary function. When setting $\lambda^a(x) = 0$, Eq. (A14) will be reduced to the constraint condition on the gauge group (the ghost equation) which is used to determine the $\theta^a(x)$ as a functional of the vector potential $A_\mu^a(x)$. However, when the constraint condition is incorporated into the action by the Lagrange undetermined multiplier method to give the ghost term in the generating functional, the $\theta^a(x)$ should be treated as arbitrary function according to the spirit of Lagrange multiplier method. That is why we may use Eq. (A16) to determine the functions $\theta^a(x)$ in terms of the function $\lambda^a(x)$ . From Eq. (A14), we solve

$$\theta^a(x) = \int d^4x(M^{-1})^{ab}(x-y)\lambda^b(y) \tag{A15}$$

Upon substituting the above expression into Eq. (A13) and then taking derivative of Eq. (A13) with respect to $\lambda^a(x)$, we obtain

$$\begin{gathered} \frac{1}{N}\int D(A)\triangle_F[A]\int d^4y[J^{b\mu}(y) + M^2A^{b\mu}(y) \\ -\frac{1}{\alpha}\partial_y^\nu A_\nu^b(y)\partial_y^\mu] D_\mu^{bc}(y)(M^{-1})^{ca}(y-x)\exp\{iS+ \\ i\int d^4x[-\frac{1}{2\alpha}(\partial^\mu A_\mu^a)^2 + J^{a\mu}A_\mu^a]\} \\ = 0 \end{gathered} \tag{A16}$$

According to the expression denoted in Eq. (2.4) and the identity $f^{bcd}A^{c\mu}A_\mu^d = 0$, it is easy to see

$$A^{b\mu}(y)D_\mu^{bc}(y)(M^{-1})^{ca}(y-x) = A^{b\mu}(y)\partial_\mu^y(M^{-1})^{ba}(y-x) \tag{A17}$$

By making use of the relation in Eq. (A10), the definition in Eq. (A3) and the equation in Eq. (A12), we deduce

$$\begin{gathered} \frac{1}{\alpha}\partial_y^\nu A_\nu^b(y)\partial_y^\mu D_\mu^{bc}(y)(M^{-1})^{ca}(y-x) \\ = \frac{1}{\alpha}\partial^\nu A_\nu^b(y)\delta^4(x-y) - M^2\partial_y^\nu A_\nu^b(y)(M^{-1})^{ba}(y-x) \end{gathered} \tag{A18}$$

On inserting Eqs. (A17) and (A18) into Eq. (A16), we obtain an identity which is exactly identical to that given in Eq. (A11) although in the above derivation, we started from the generating functional without containing the ghost field functions and the BRST-sources and , therefore, the derivation does not concern the nilpotency of the composite field functions appearing in the BRST-source terms. This fact indicates that the W-T identities derived in section 3 are correct and hence the procedure of introducing the BRST-invariant source terms into the generating functional is completely reasonable.

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## XIV. FIGURE CAPTIONS

Fig. (1): The one-loop gluon self-energy. The solid, wavy and dashed lines represent the free quark, gluon and ghost particle propagators respectively.
Fig. (2): The one-loop ghost particle self-energy. The lines represent the same as in Fig. (1).
Fig. (3): The one-loop ghost-gluon vertices . The lines mark the same as in Fig. (1).

Fig. (4): The one-loop effective coupling constants $\alpha_R(\lambda)$ given by the timelike momentum space subtraction. The solid and dashed lines represent the coupling constants given by the massive QCD and the massless QCD, respectively. The dotted line denotes the coupling constant given in the minimal subtraction scheme.

Fig. (5): The one-loop effective coupling constants $\alpha_R(\lambda)$ given by the spacelike momentum space subtraction. The solid and dashed lines represent the coupling constants given by the massive QCD and the massless QCD, respectively. The dotted line denotes the coupling constant given in the minimal subtraction scheme. The dashed-dotted line represents the one-loop effective coupling constants $\alpha_R(\lambda)$ given by the spacelike momentum subtraction for which the gluon mass is taken to be a smaller value.

Fig. (6): The one-loop effective coupling constants $\alpha_R(\lambda)$ given by the timelike momentum space subtraction. The solid, dashed and dotted lines represent the coupling constants given by taking the quark flavor $N_f = 2, 3$ and 4 respectively.

Fig. (7): The one-loop effective gluon masses $M_R(\lambda)$. The solid and the dashed lines represent the effective masses given in the timelike and spacelike subtractions respectively.

Fig. (8): The one-loop quark-gluon vertices. The lines represent the same as in Fig. (1).

Fig. (9): The one-loop quark-ghost particle vertices. The lines represent the same as in Fig. (1).

Fig. (10): The one-loop quark self-energy. The lines represent the same as in Fig. (1).

Fig. (11): The one-loop effective quark masses $m_R(\lambda)$ given by the timelike momentum space subtraction. The solid and the dashed lines represent the effective masses given by the massive QCD and massless QCD, respectively.

Fig. (12): The one-loop effective quark masses $m_R(\lambda)$ given by the spacelike momentum space subtraction. The solid and the dashed lines represent respectively the real part and imaginary part of the effective quark mass which is given by the massive QCD. The dotted line shows the real part of the effective quark mass which is given by the massless QCD.